Product of a Grammar with an n-Gram Model for Statistical Machine Translation

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Outline

Modelling and Training

translation function h: $F \rightarrow E$

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translation function $h\colon F \to E$ (set of hypotheses $\mathcal{H} \subseteq E^F$)

$\mathcal{M} \subseteq \mathcal{M}(E \mid F)$

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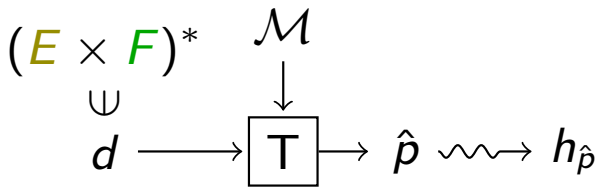

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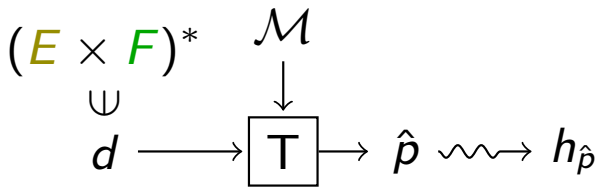

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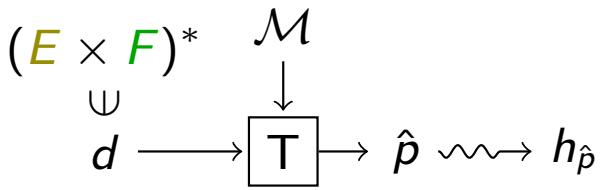

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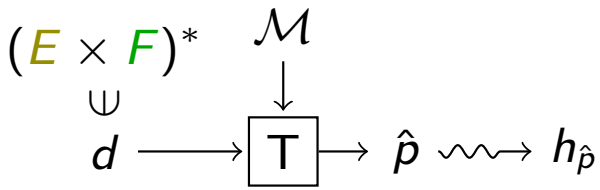

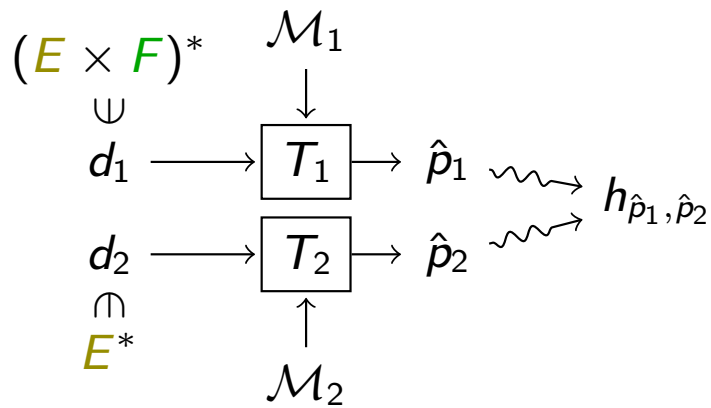

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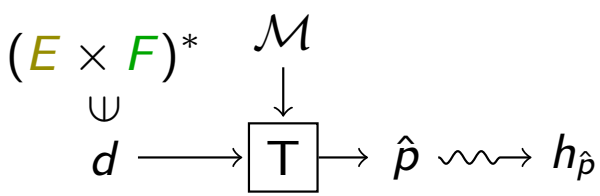

source-channel approach

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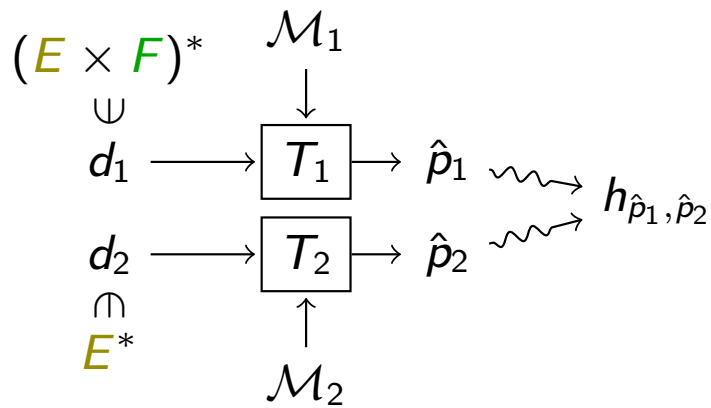

Outline

Decoding

source-channel approach

$$h_{p_1,p_2}(f) = \text{argmax}_{e \in E} \underbrace{p_1}_{\in \mathcal{M}_1}(f \mid e) \cdot \underbrace{p_2}_{\in \mathcal{M}_2}(e)$$

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- $\mathcal{M}_2$ . . . n-gram model

Outline

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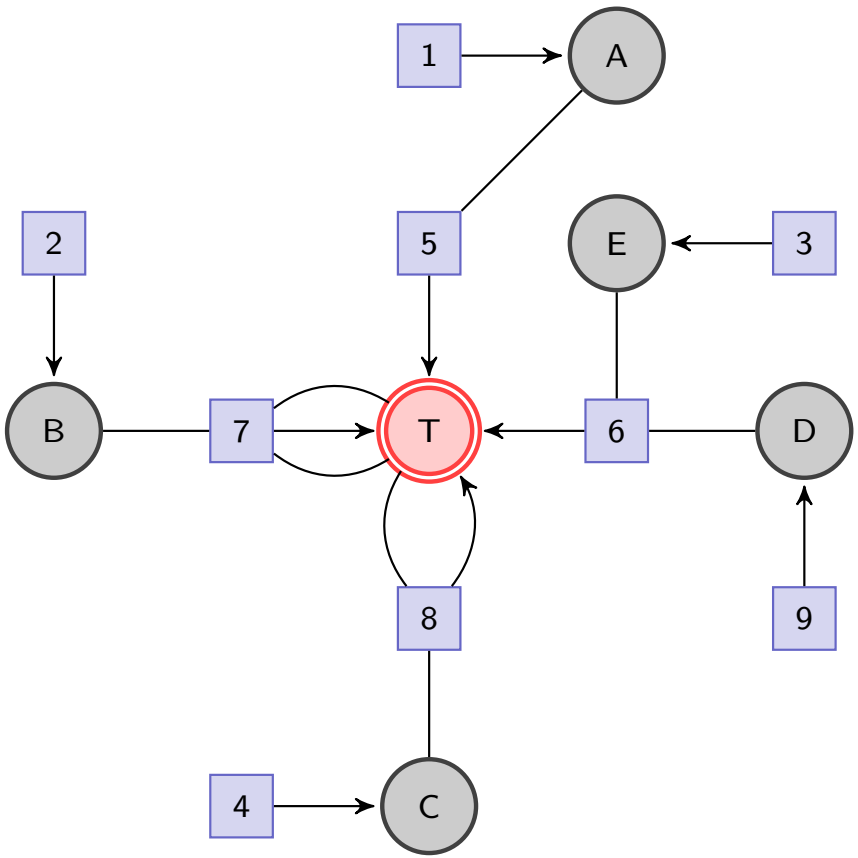
- RTG (shown as hypergraph)



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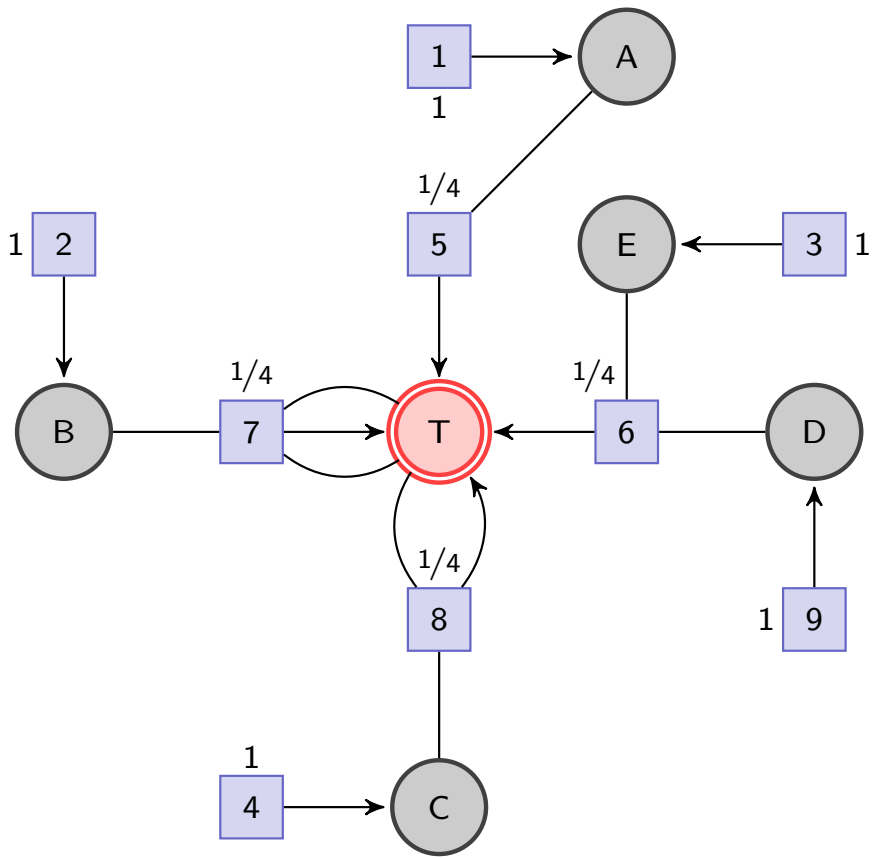

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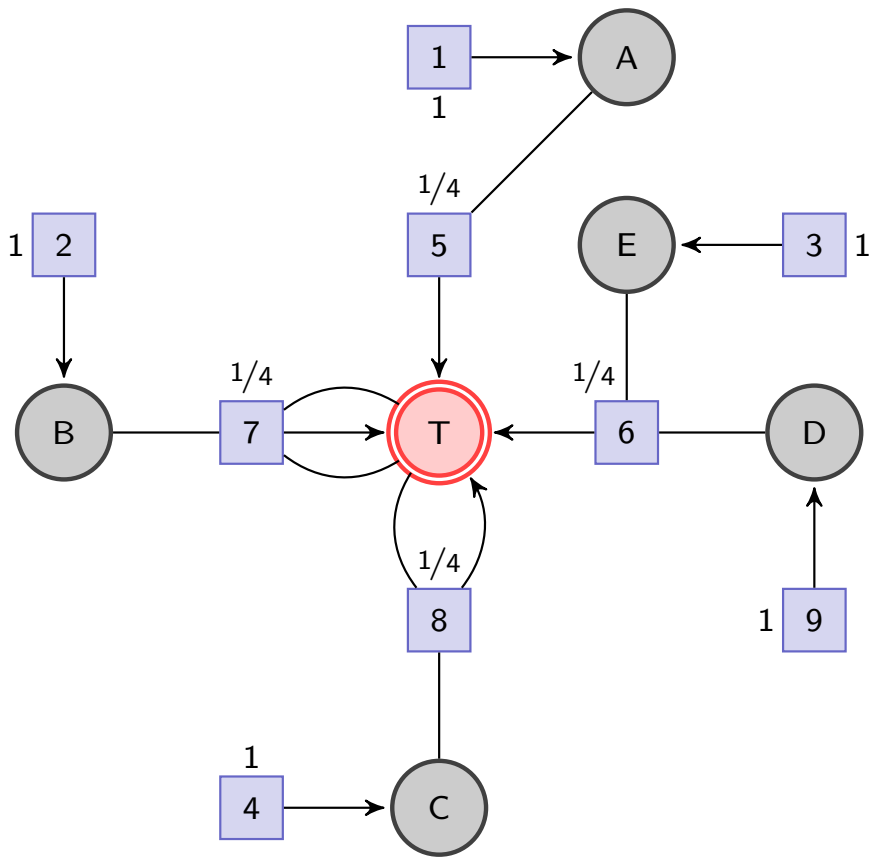

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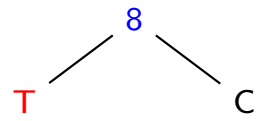

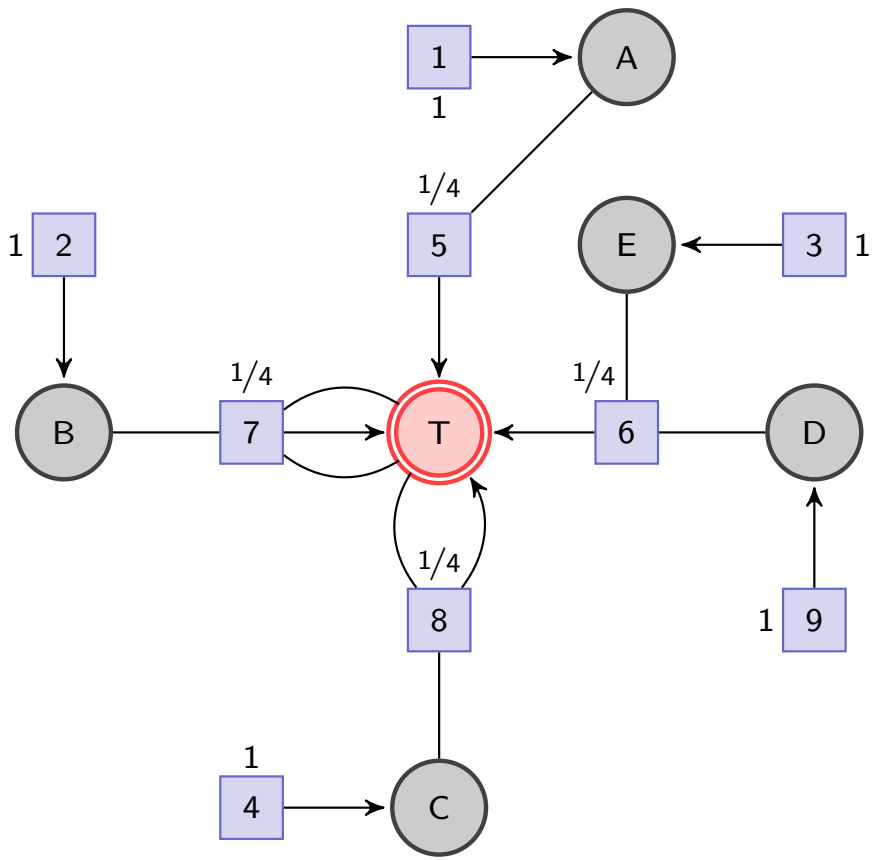

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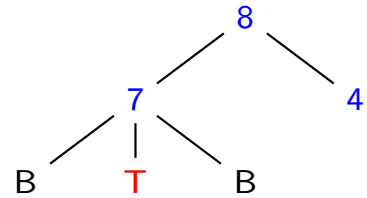

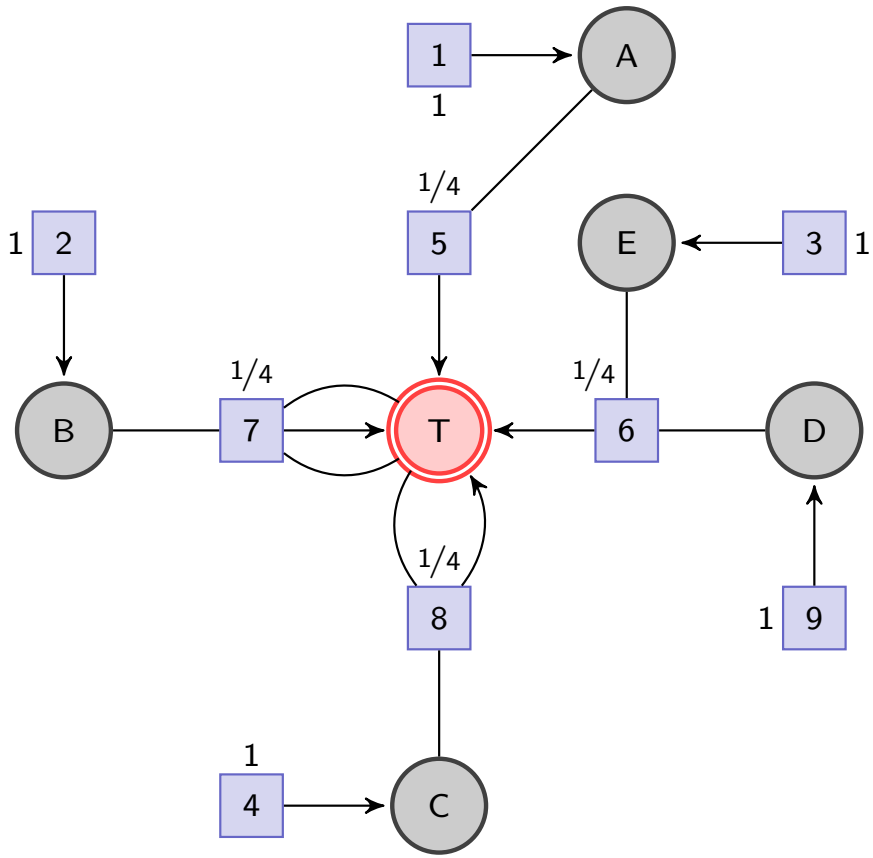

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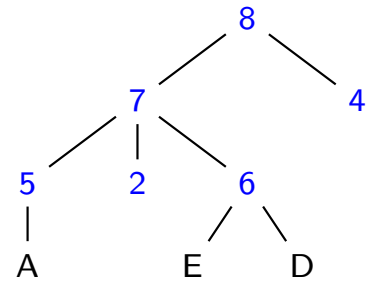

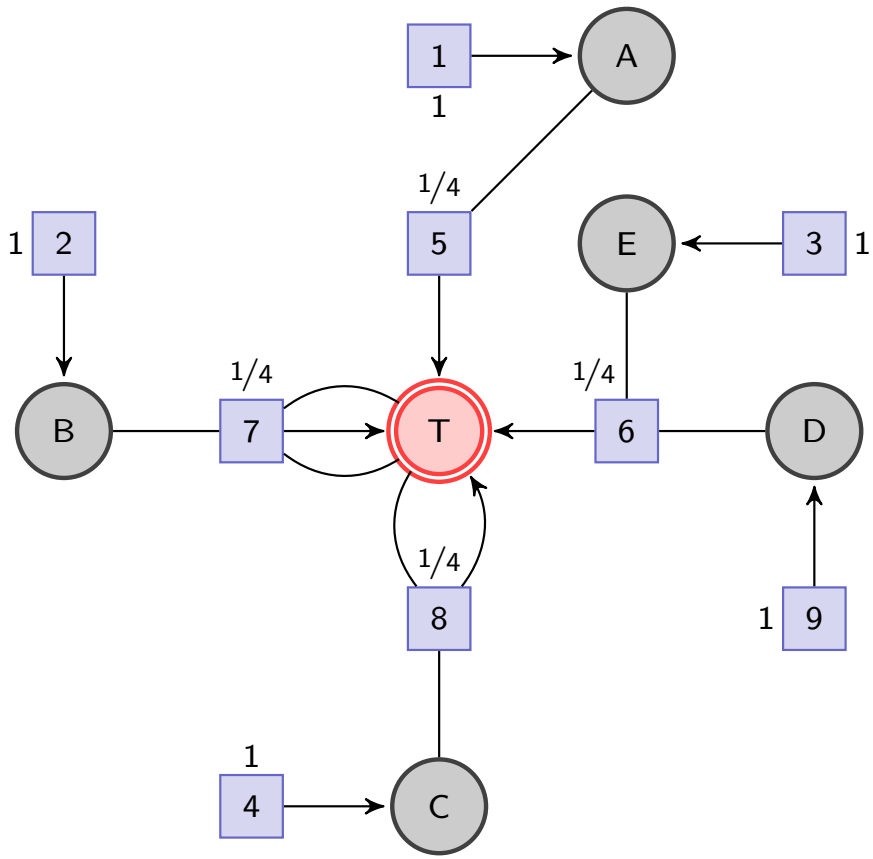

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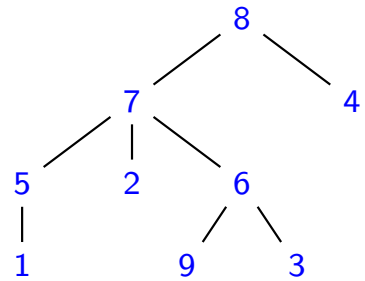

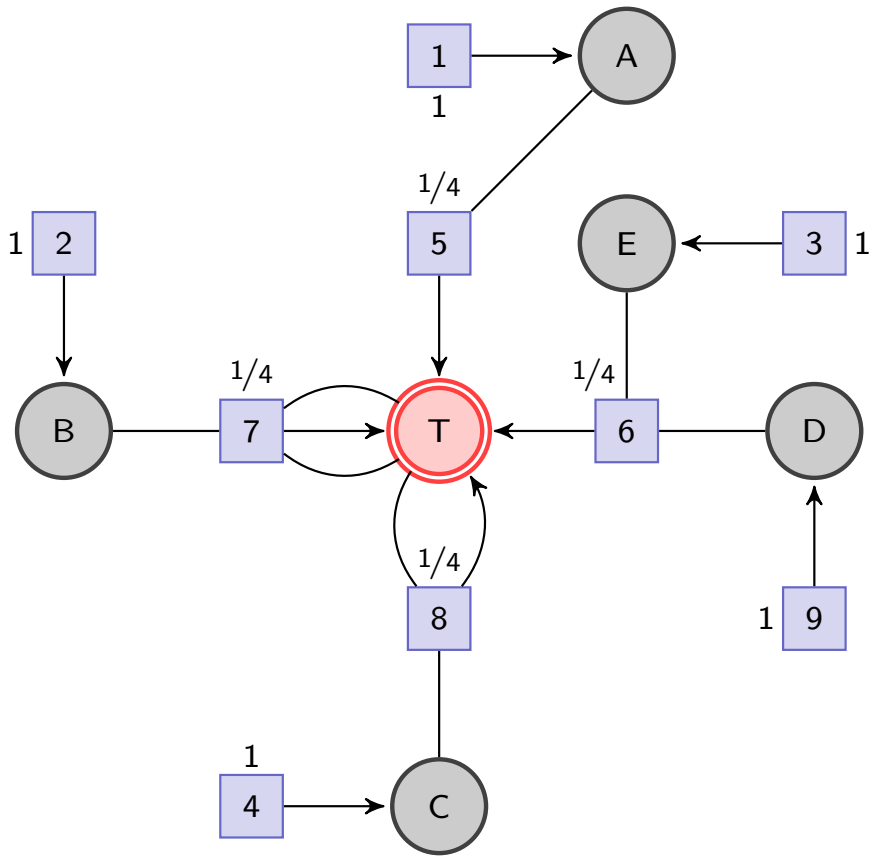

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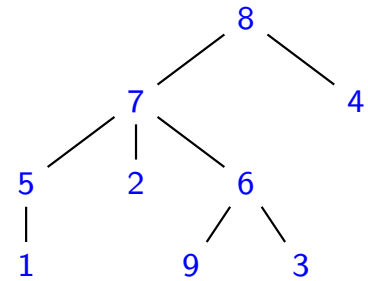

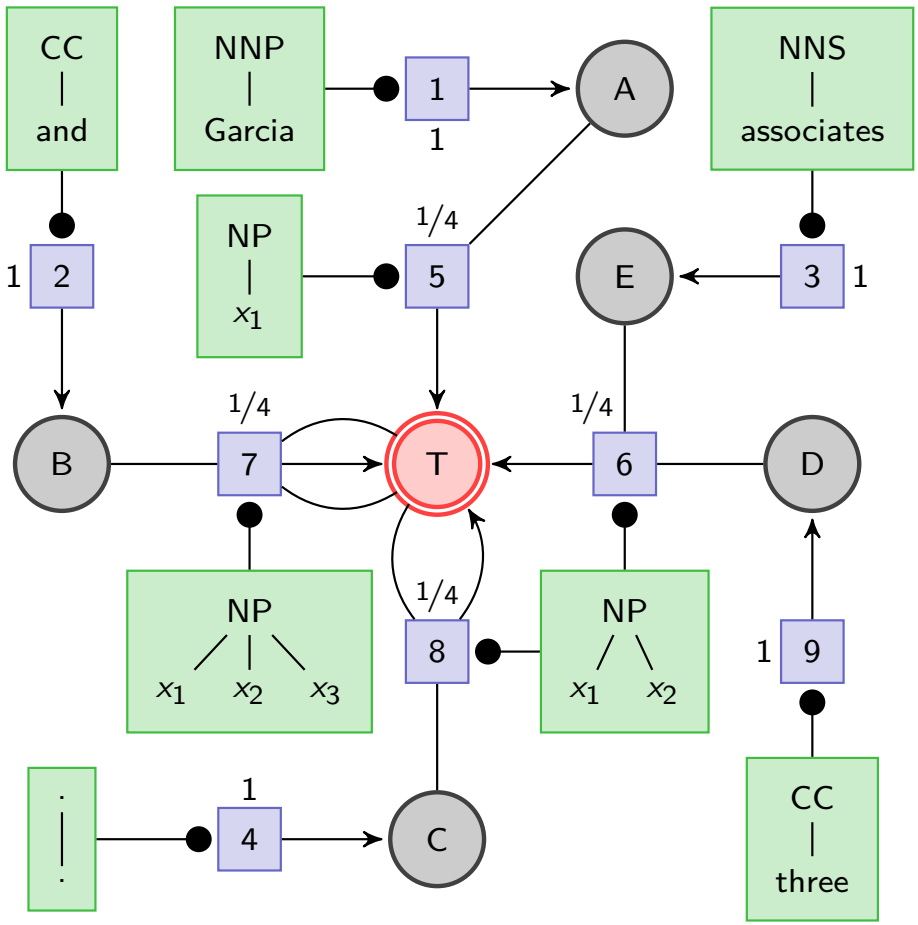

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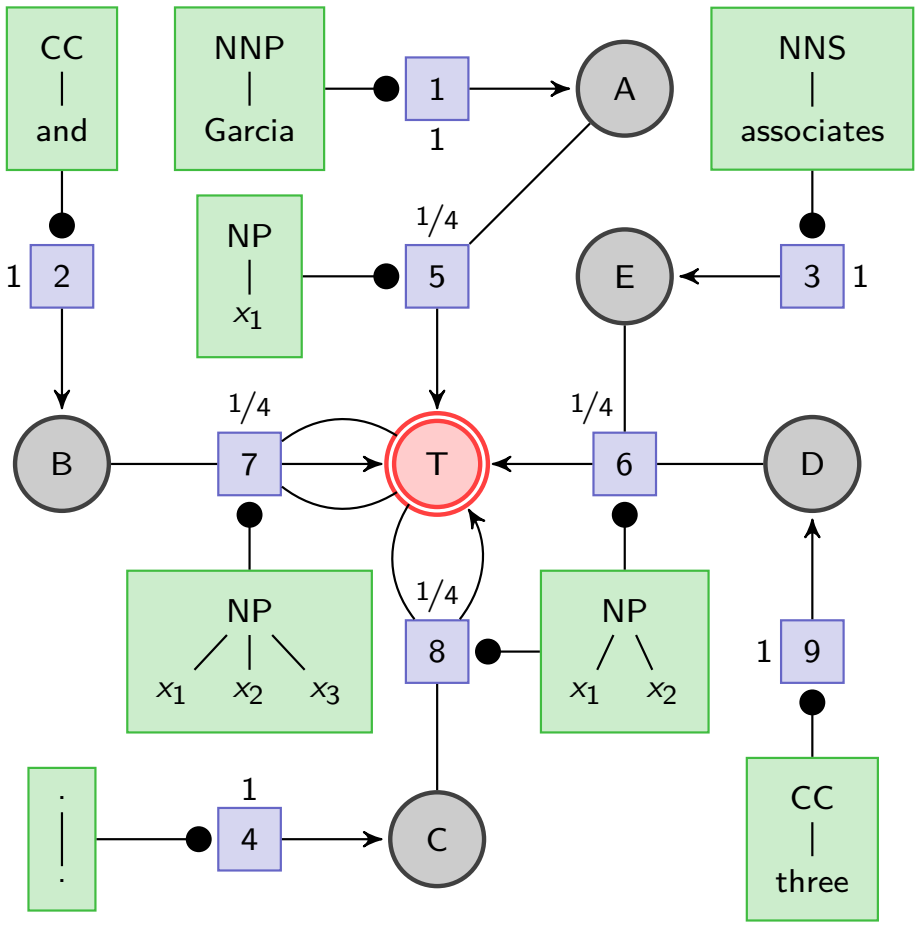

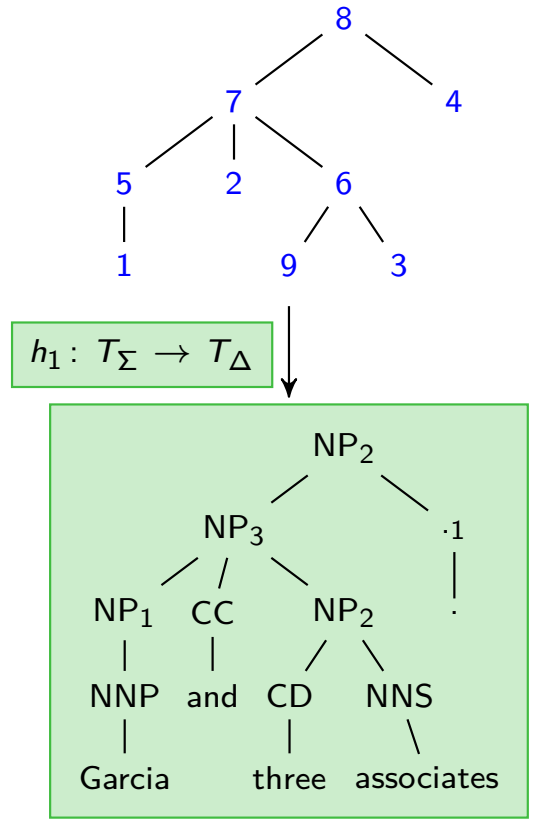

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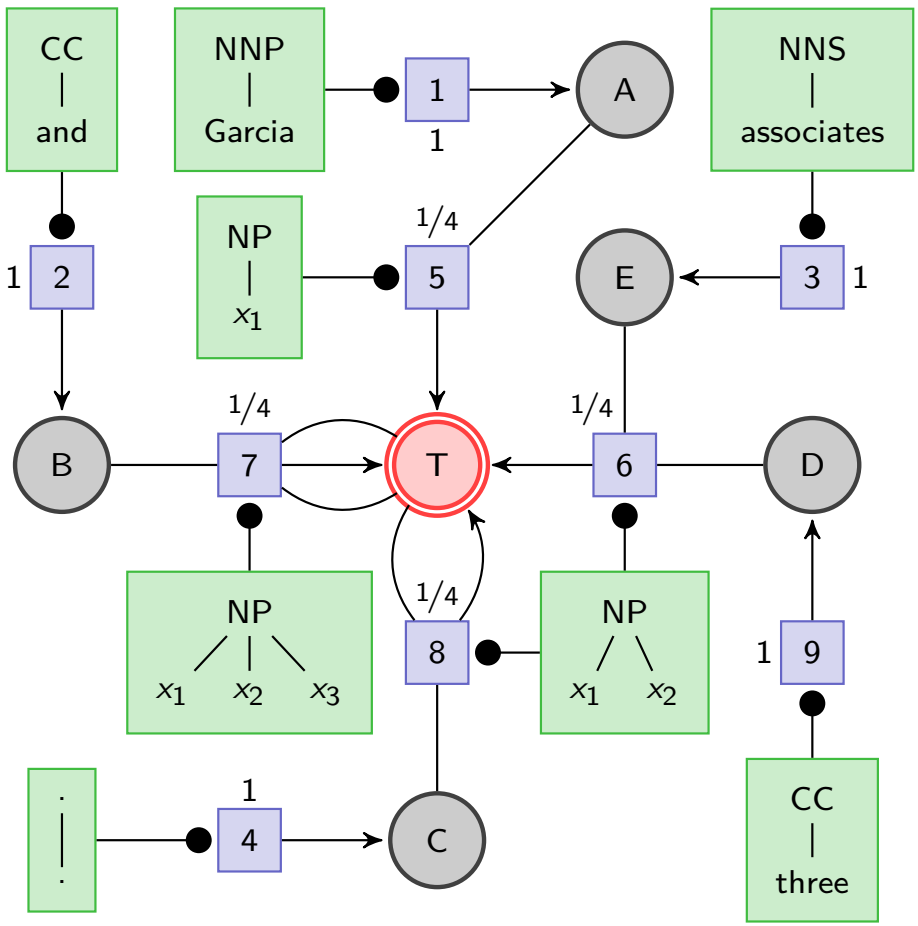

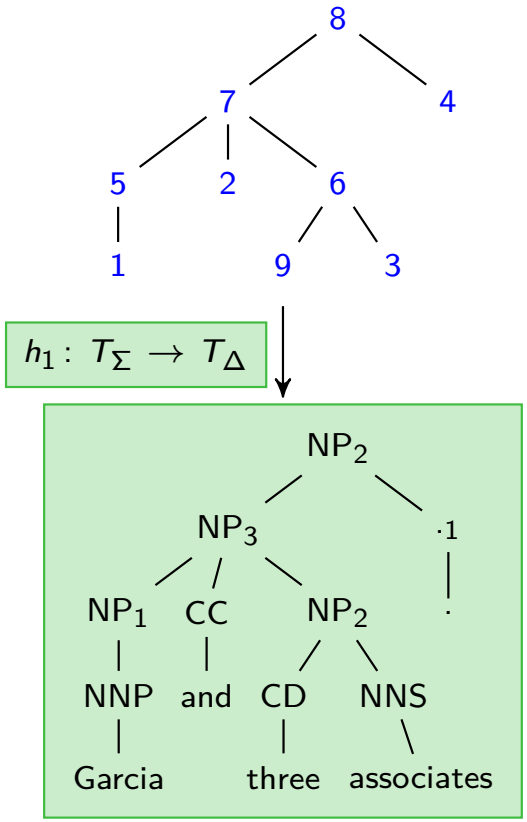

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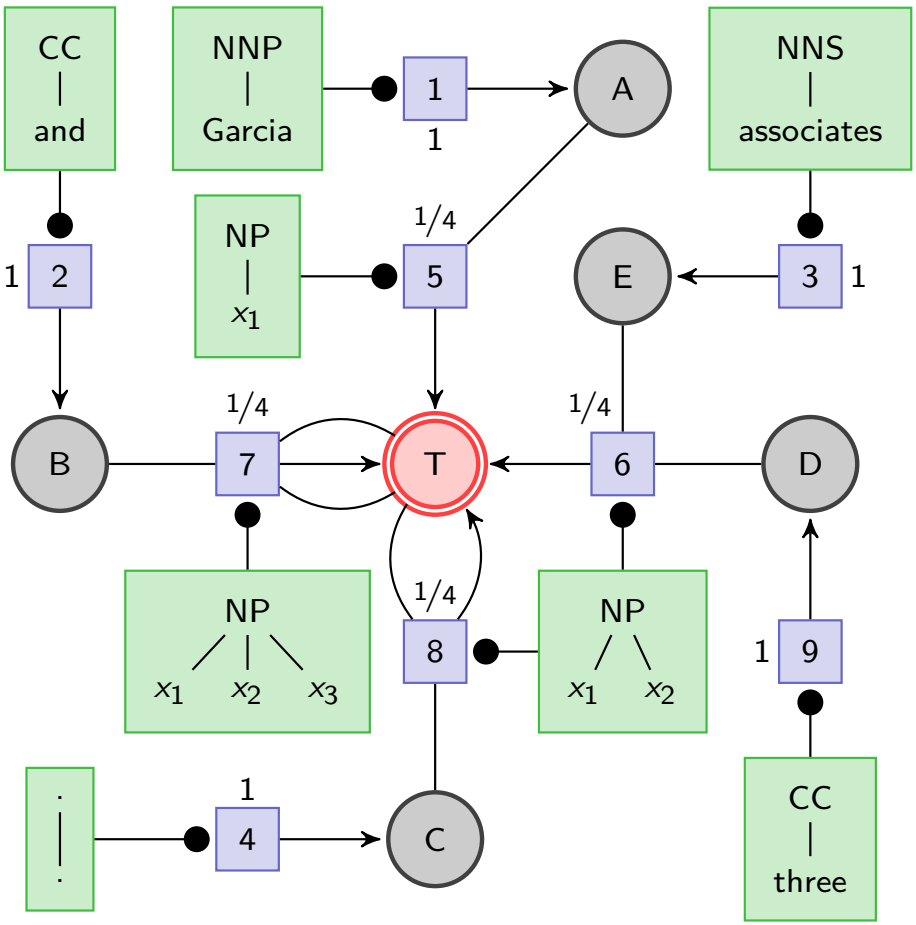

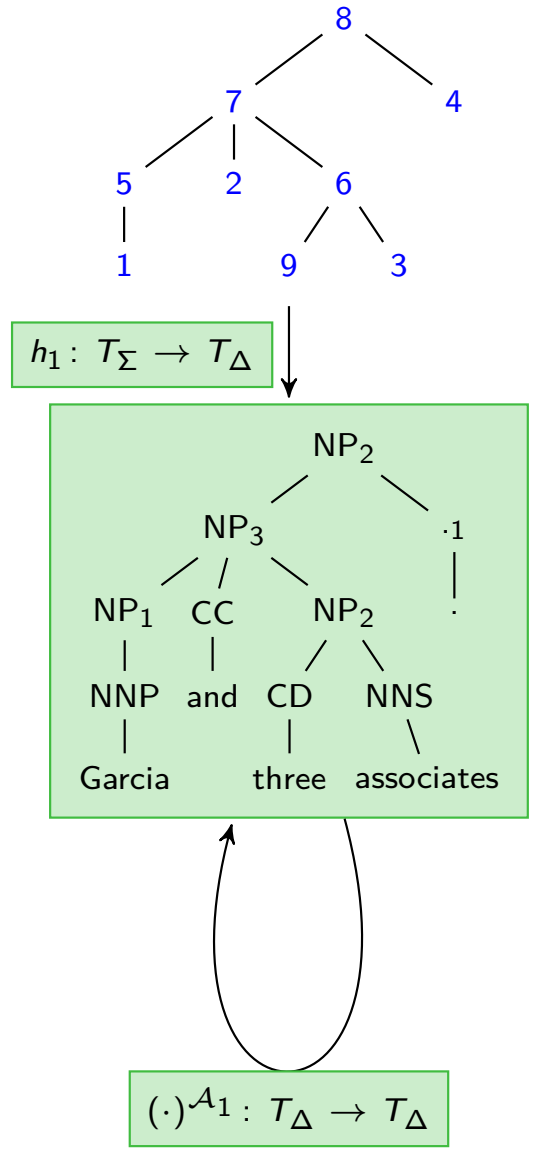

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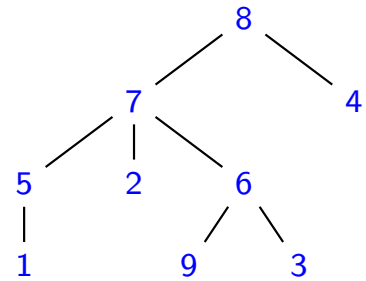

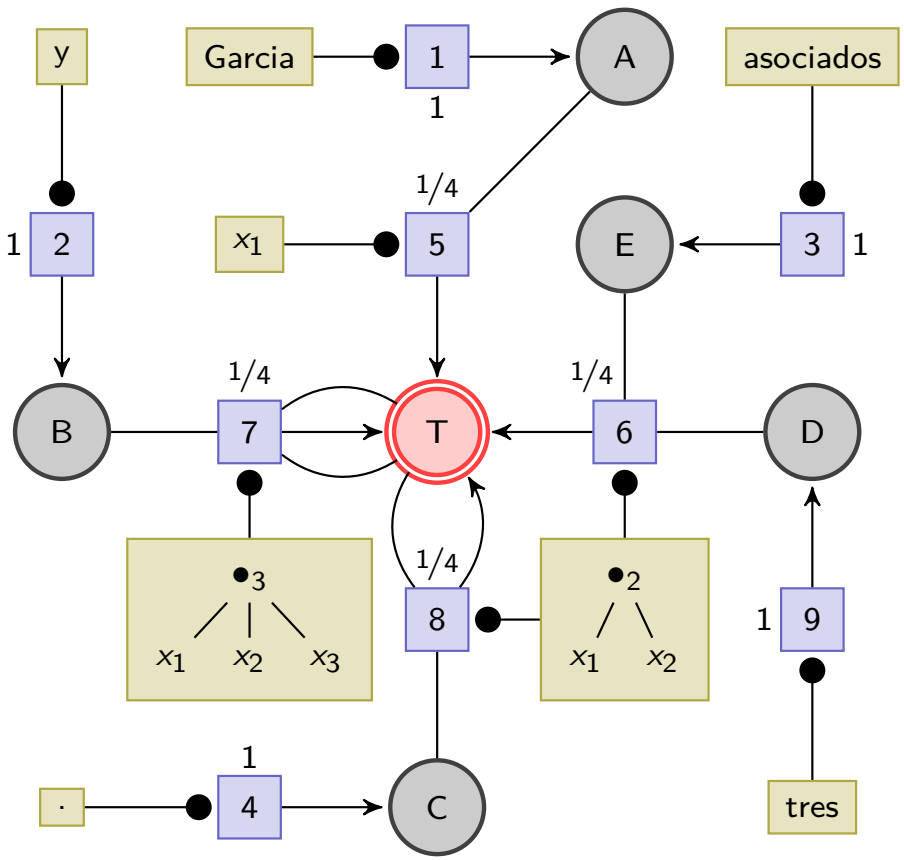

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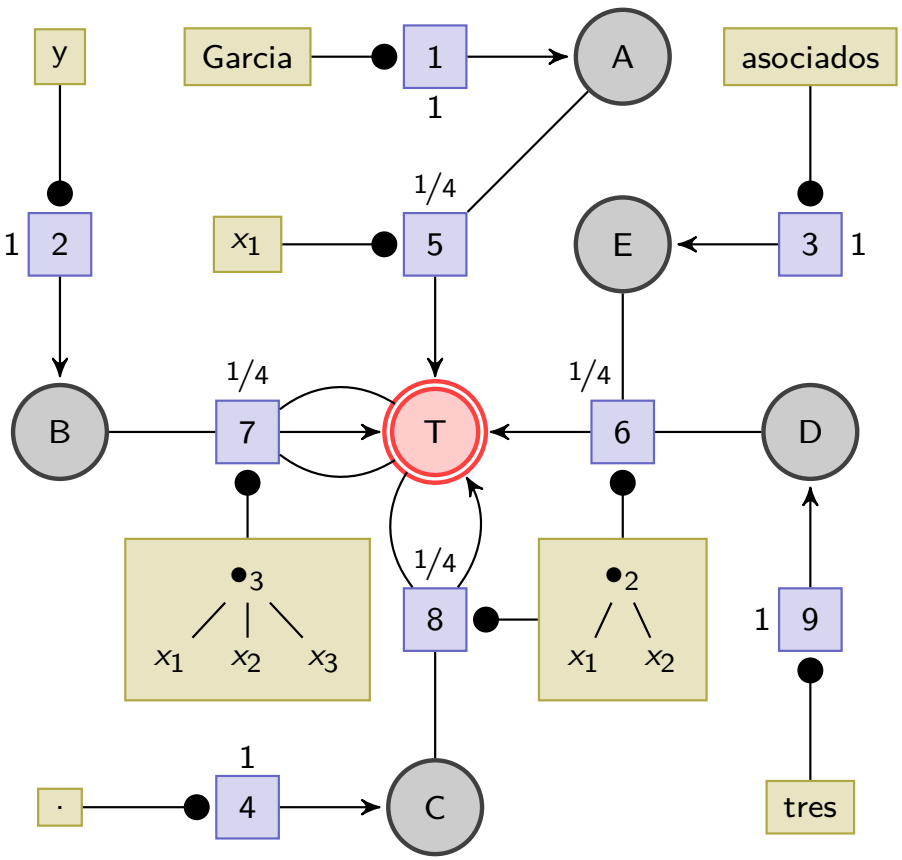

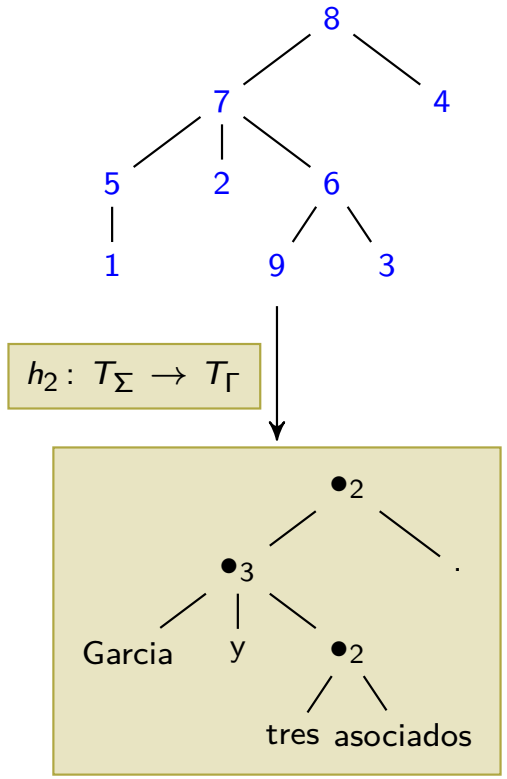

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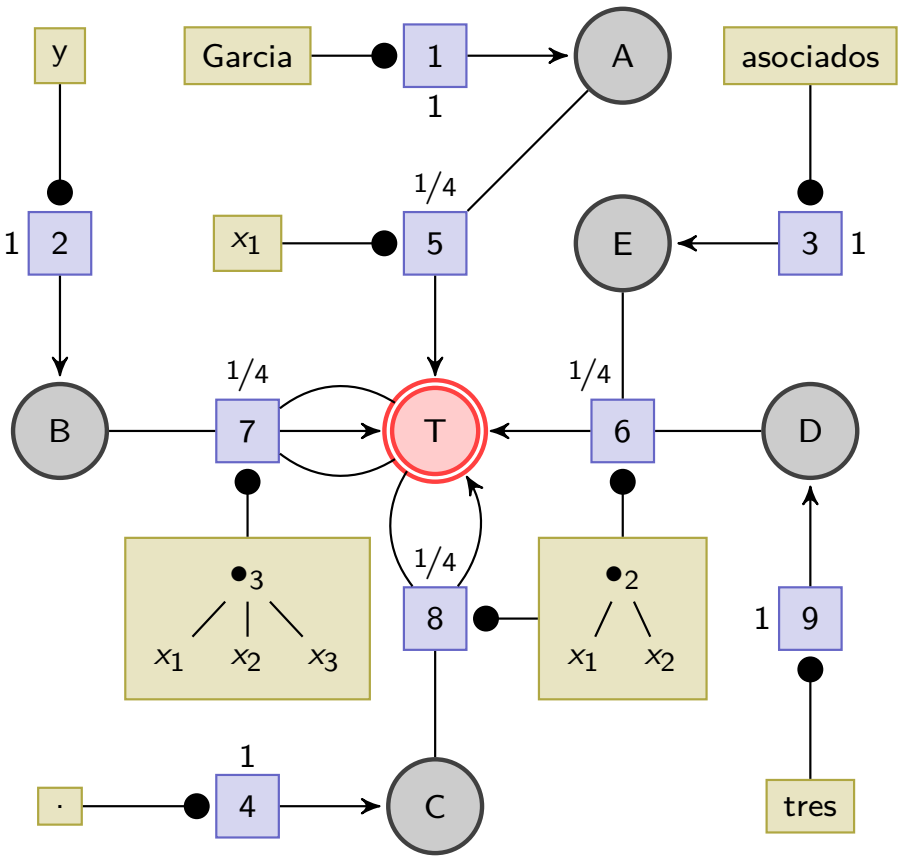

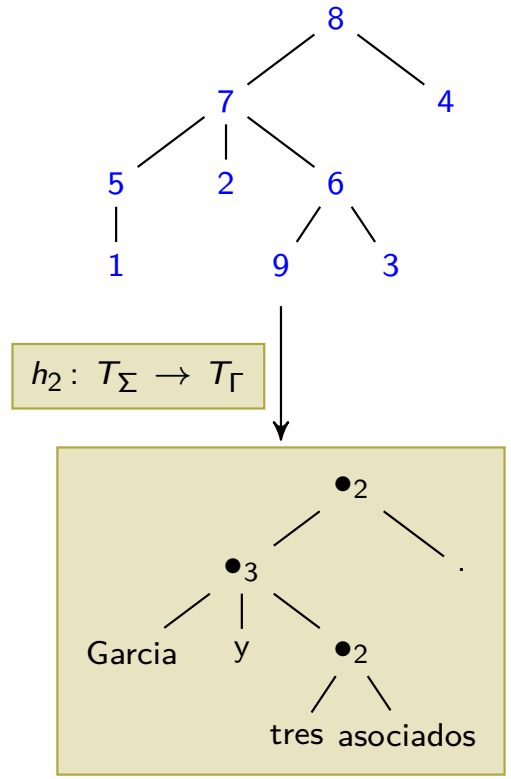

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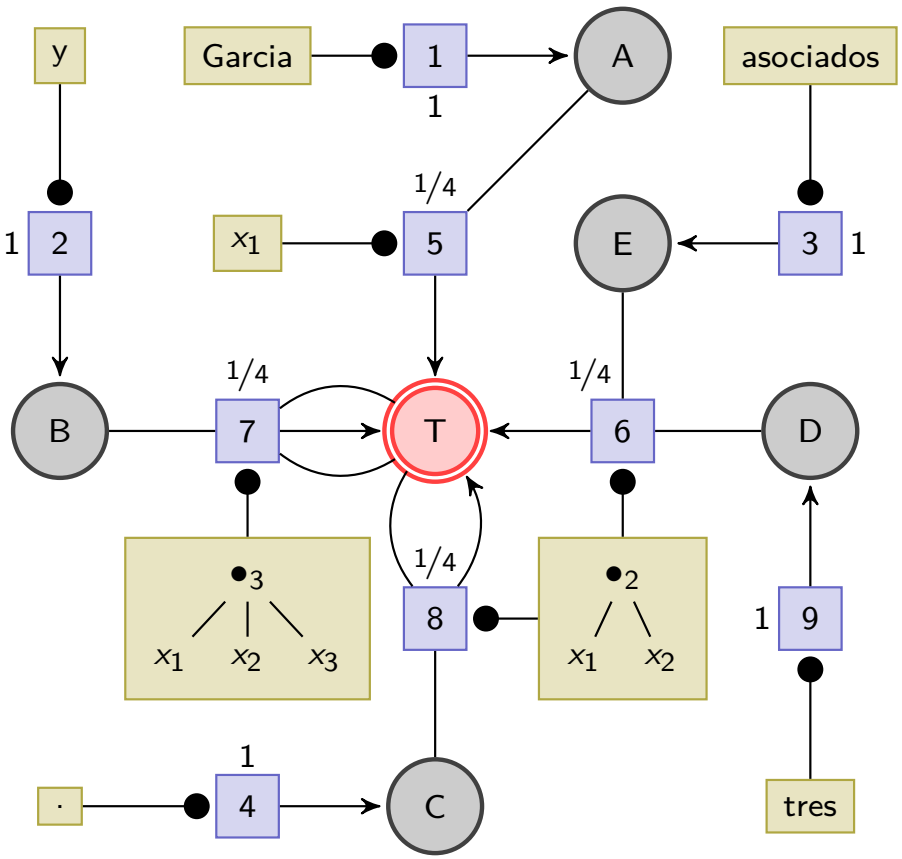

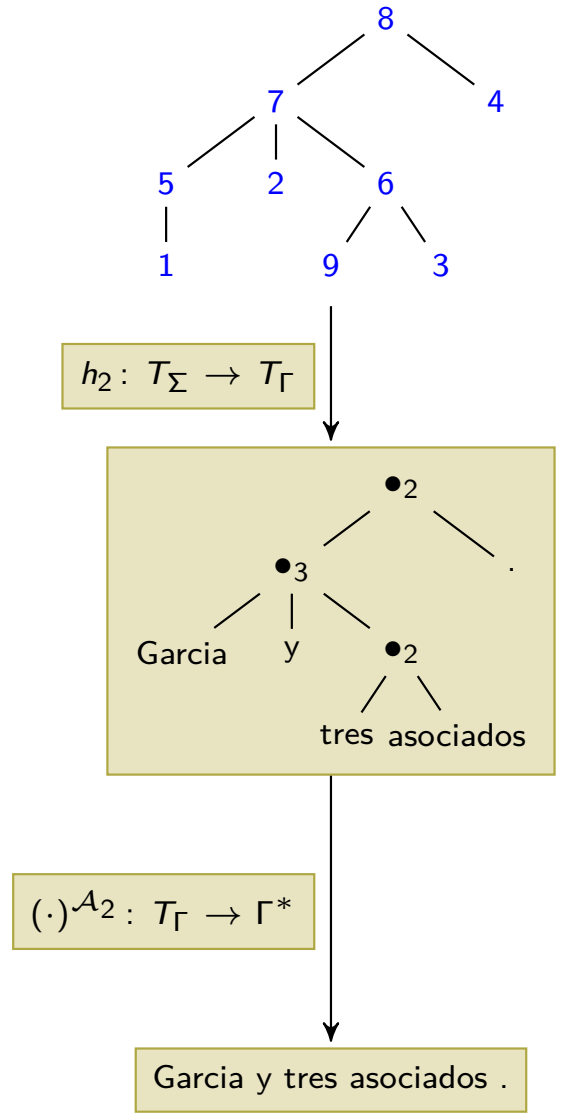

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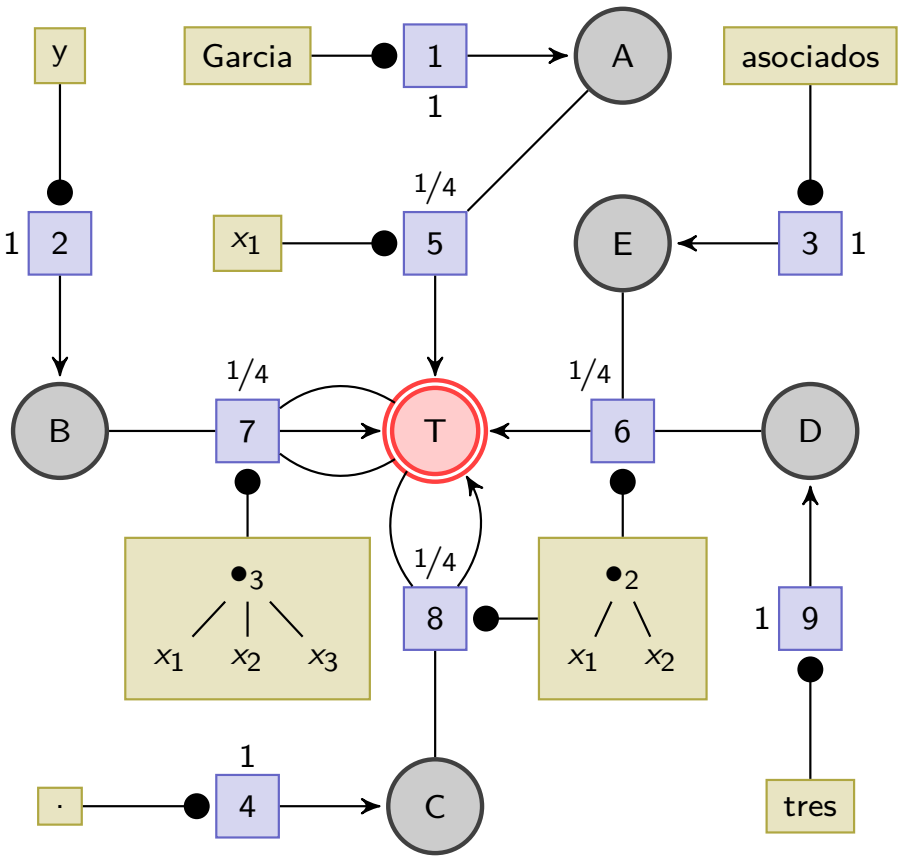

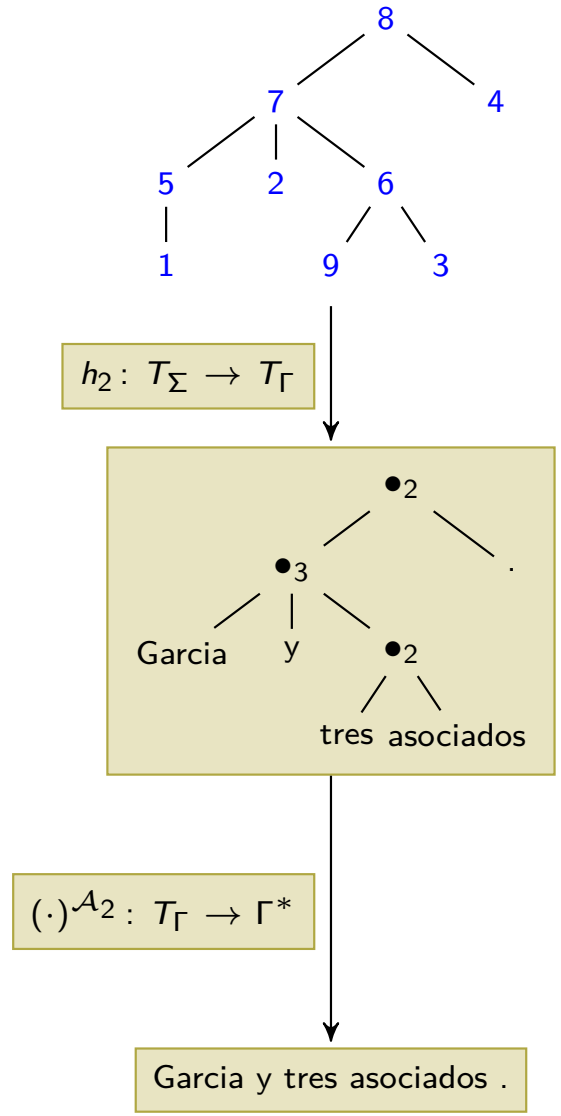

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- assigns weight to every $\underbrace{\text{sequence of 3 symbols}}_{= \text{3-gram}}$

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n-Gram Model

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$N(w) =$ $2/5$ $1/3$ $1/4$

n-Gram Model

example: 3-gram model ($n = 3$)

- scores a sequence of symbols
- assigns weight to every $\underbrace{\text{sequence of 3 symbols}}_{= \text{3-gram}}$
- weights are multiplied

$$w = \boxed{\text{Garcia}}\ \boxed{\text{y}}\ \boxed{\text{tres}}\ \boxed{\text{asociados}}\ \boxed{\text{.}}$$

$$N(w) = {}^{2}/_{5} \cdot {}^{1}/_{3} \cdot {}^{1}/_{4} = {}^{1}/_{30}$$

Outline

Component Product

$$h_{p_1,p_2}(f) = \text{argmax}_{e \in E} \underbrace{p_1}_{\text{by IRTG}}(f \mid e) \cdot \underbrace{p_2}_{\text{by } n\text{-gram model}}(e)$$

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$$(\mathcal{L} \odot_i N)(\langle w_1, \ldots, w_k \rangle) = \mathcal{L}(\langle w_1, \ldots, w_k \rangle) \cdot N(w_i)$$

Results

Theorem (main result)

$$\mathit{IRTG} \odot_i \mathit{n\text{-}gram\ model} \subseteq \mathit{IRTG}$$

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Lemma (*n*-gram wRTG)

$$(\cdot)^{\mathcal{A}}; \mathit{n\text{-}gram\ model} \subseteq \mathit{wRTG}$$

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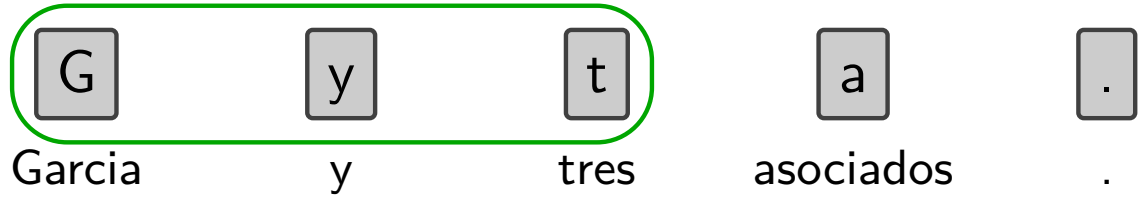
Outline

n-Gram wRTG

$w =$ 

$N(w) =$

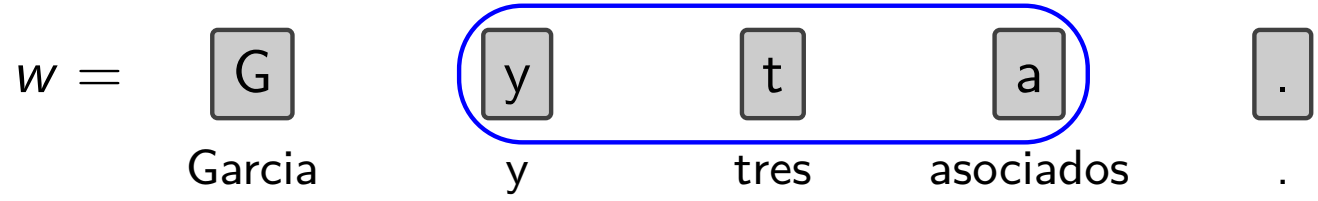
n-Gram wRTG

$w =$ 

$N(w) =$

$N(\text{Gyt})$
$2/5$

n-Gram wRTG



$$N(w) = \underset{2/5}{N(\text{Gyt})} \cdot \underset{1/3}{N(\text{yta})}$$

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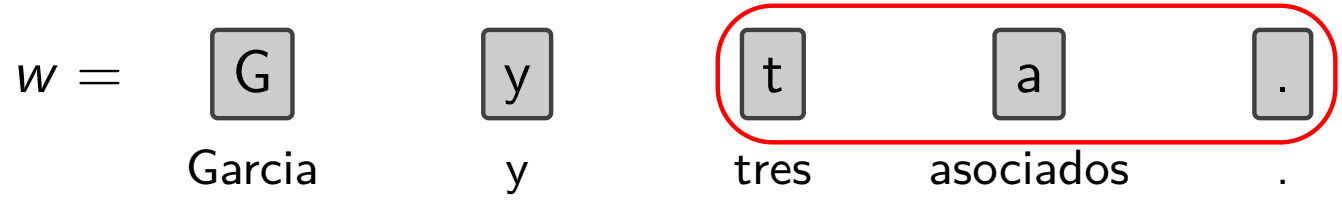

$$N(w) = \underset{N(\text{Gyt})}{2/5} \cdot \underset{N(\text{yta})}{1/3} \cdot \underset{N(\text{ta.})}{1/4}$$

n-Gram wRTG

$$w = \underset{\text{Garcia}}{\boxed{\text{G}}} \quad \underset{\text{y}}{\boxed{\text{y}}} \quad \underset{\text{tres}}{\boxed{\text{t}}} \quad \underset{\text{asociados}}{\boxed{\text{a}}} \quad \underset{\text{.}}{\boxed{\text{.}}}$$

$$N(w) = \underset{N(\text{Gyt})}{2/5} \cdot \underset{N(\text{yta})}{1/3} \cdot \underset{N(\text{ta.})}{1/4} = 1/30$$

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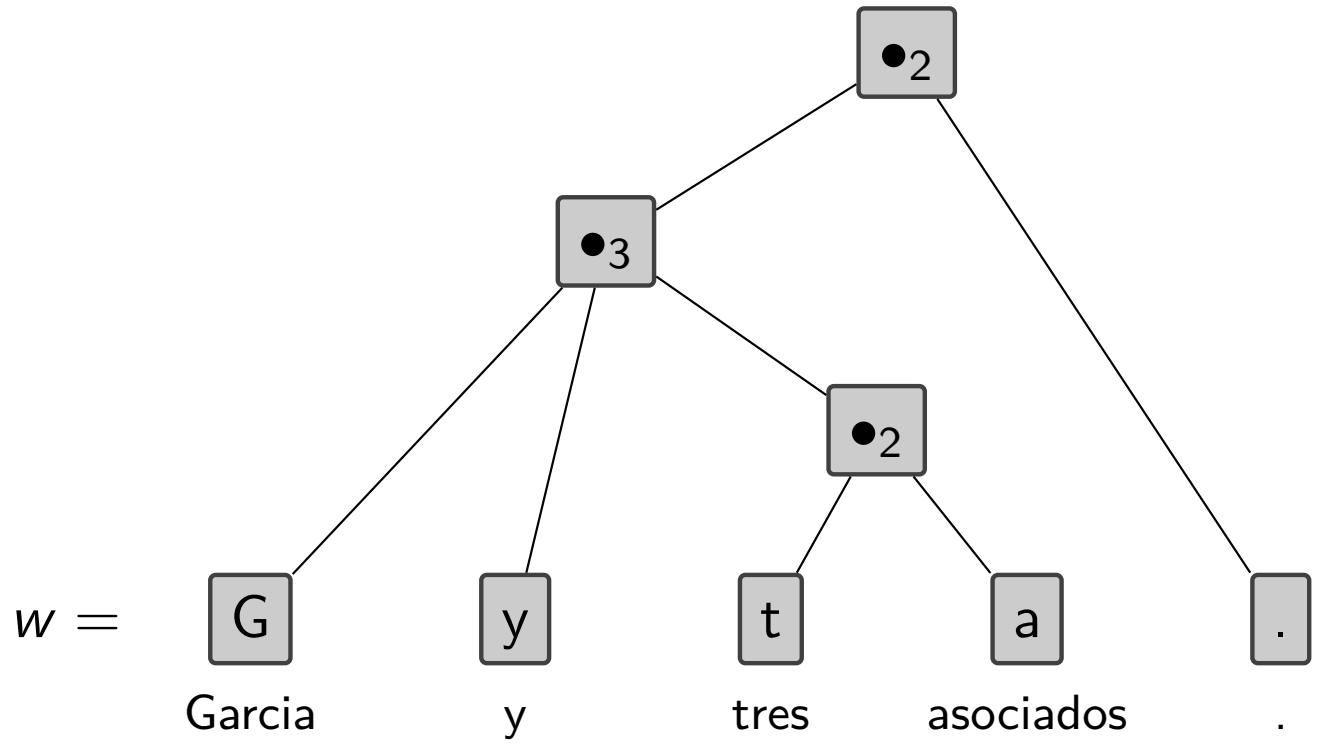

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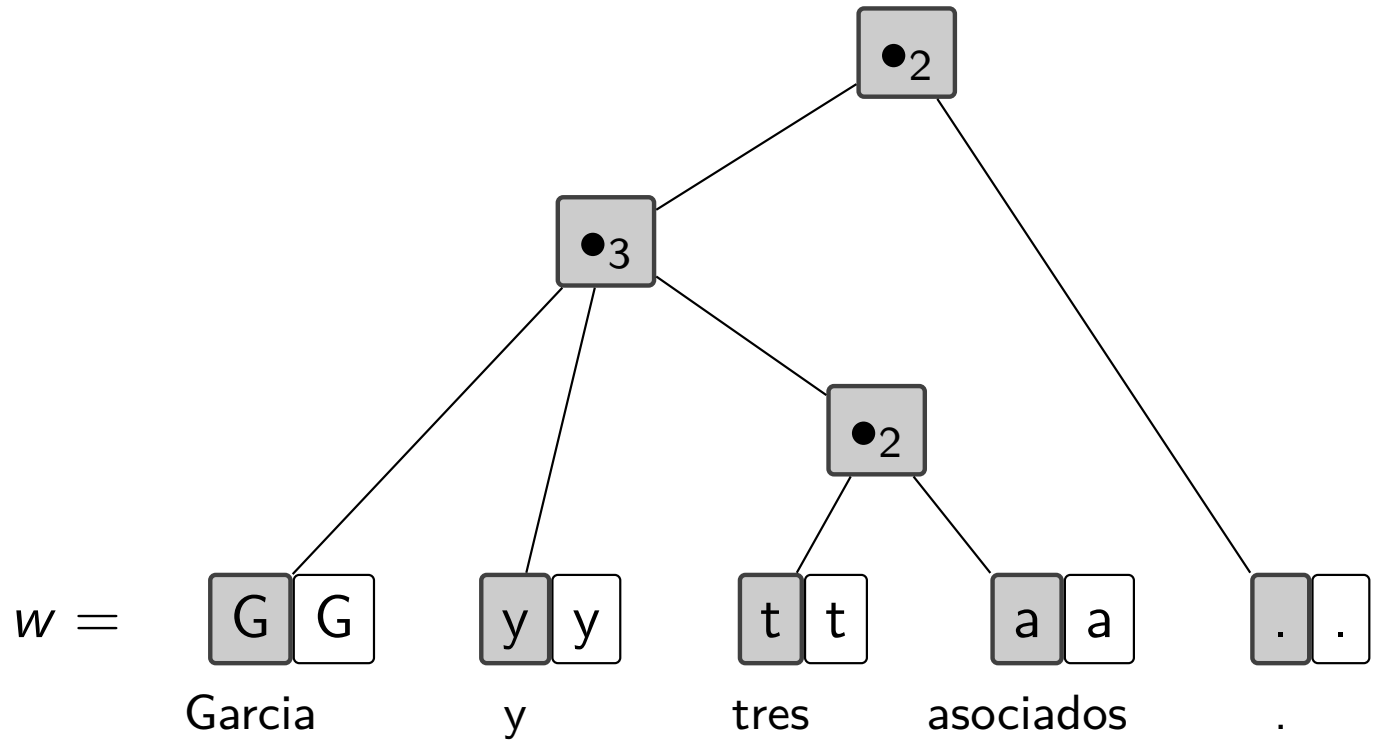

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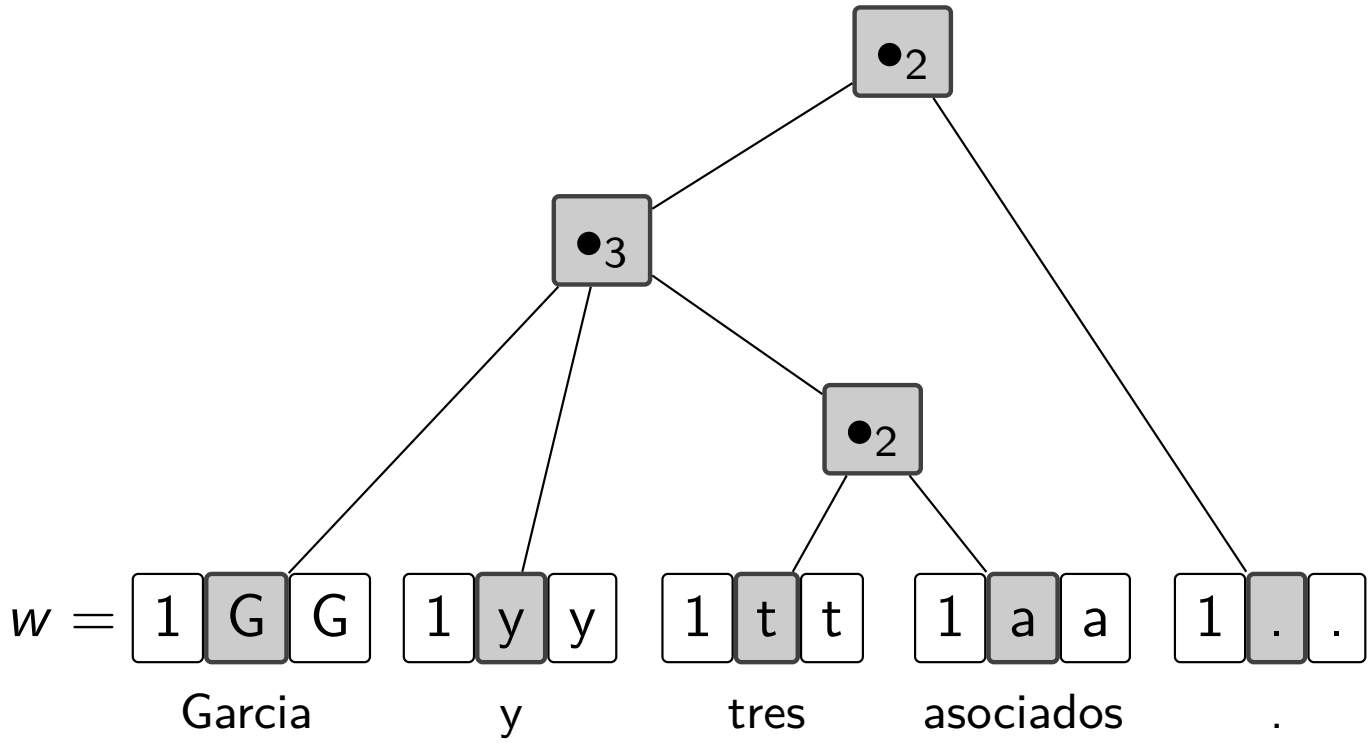

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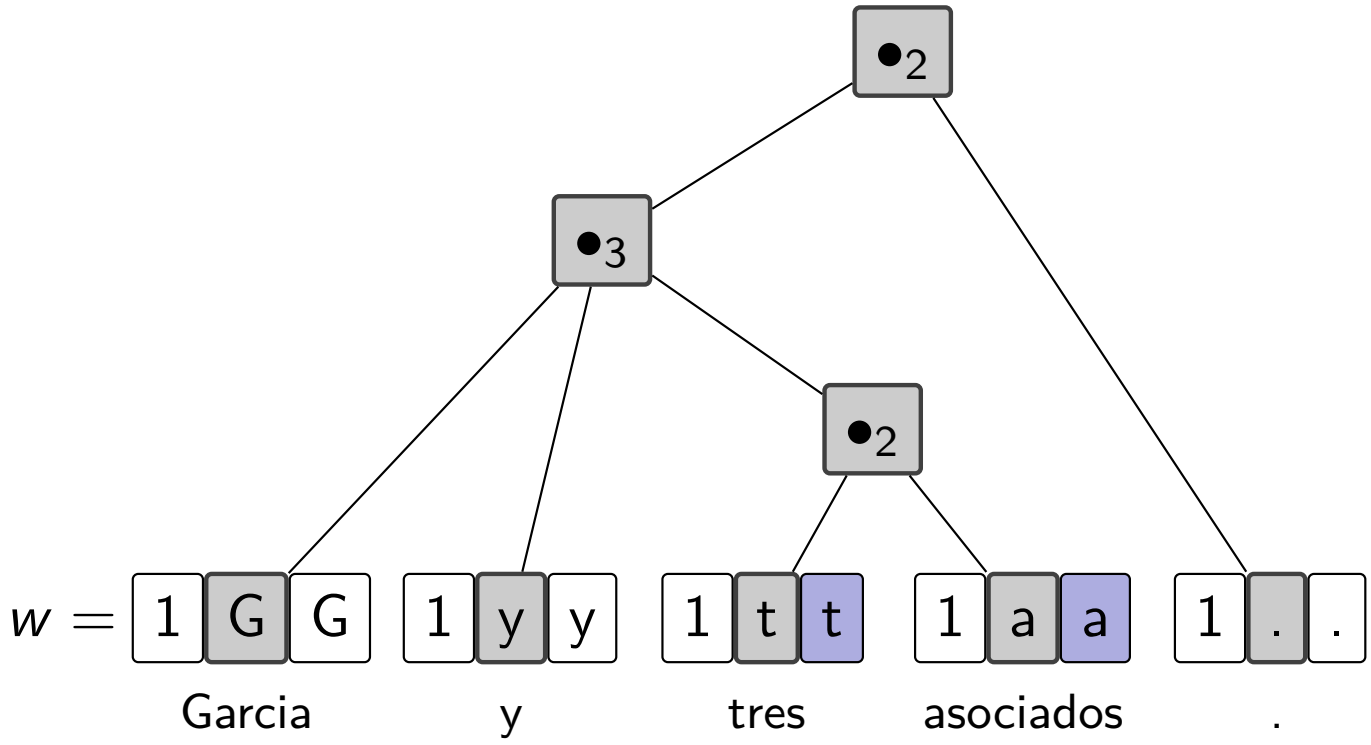

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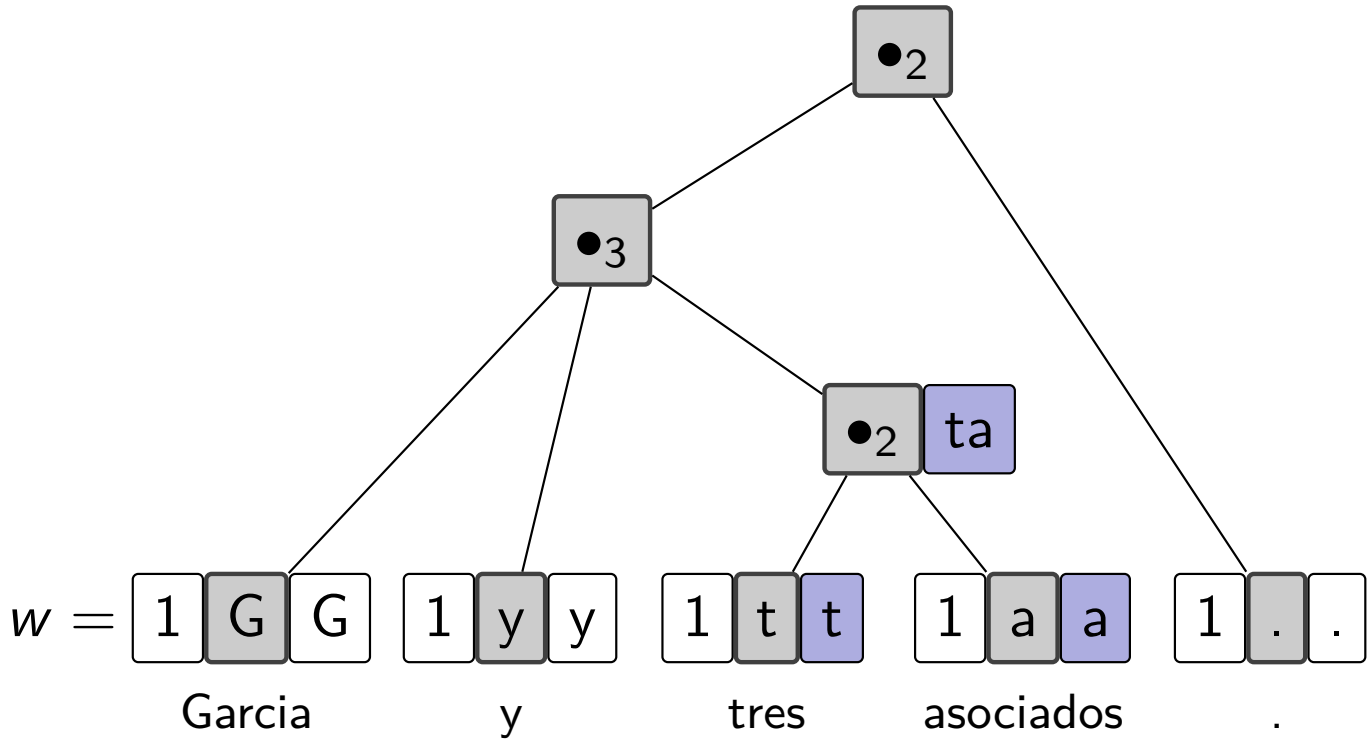

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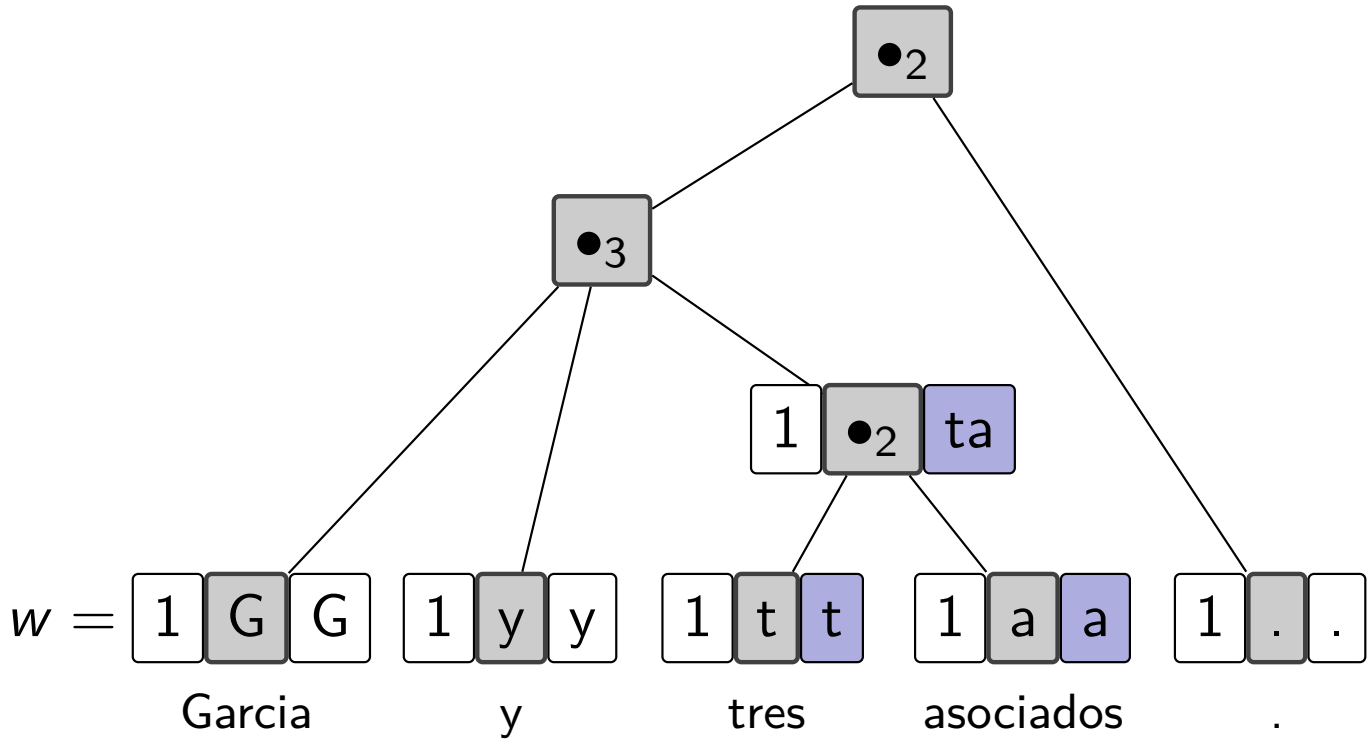

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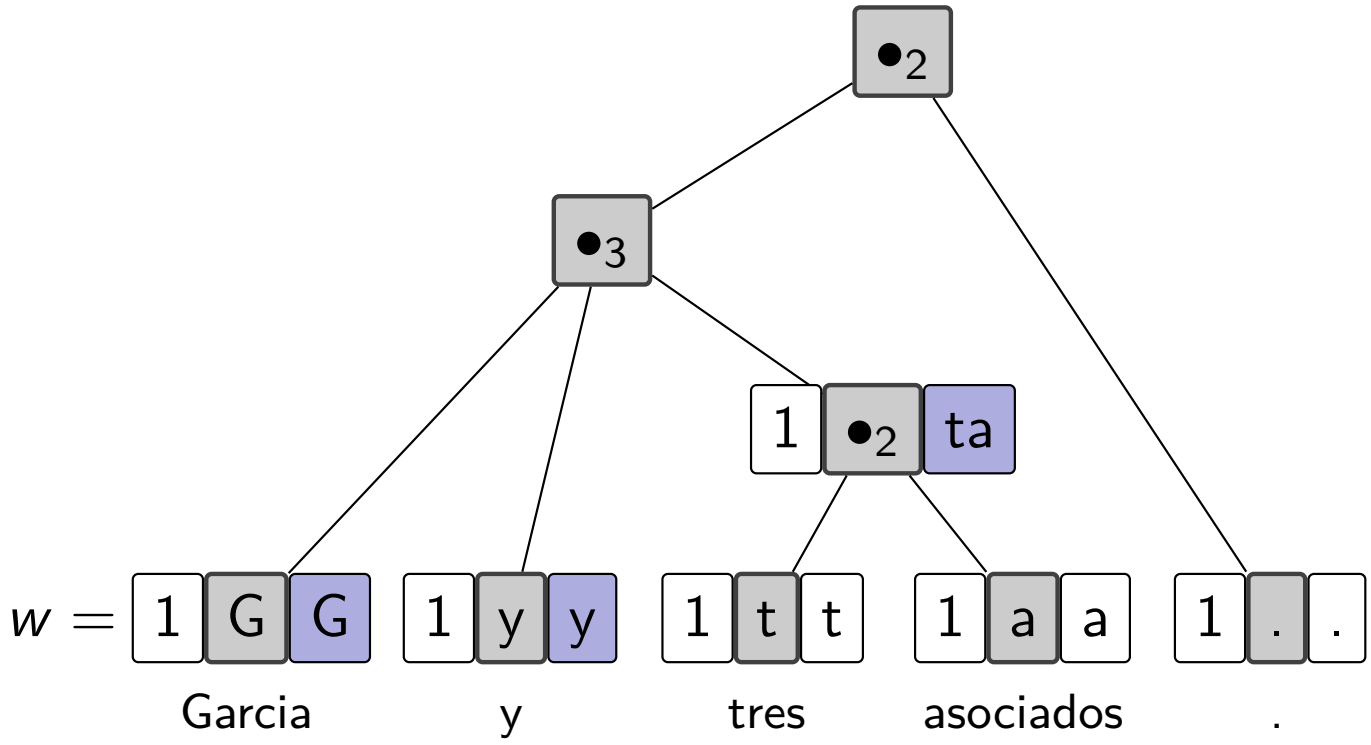

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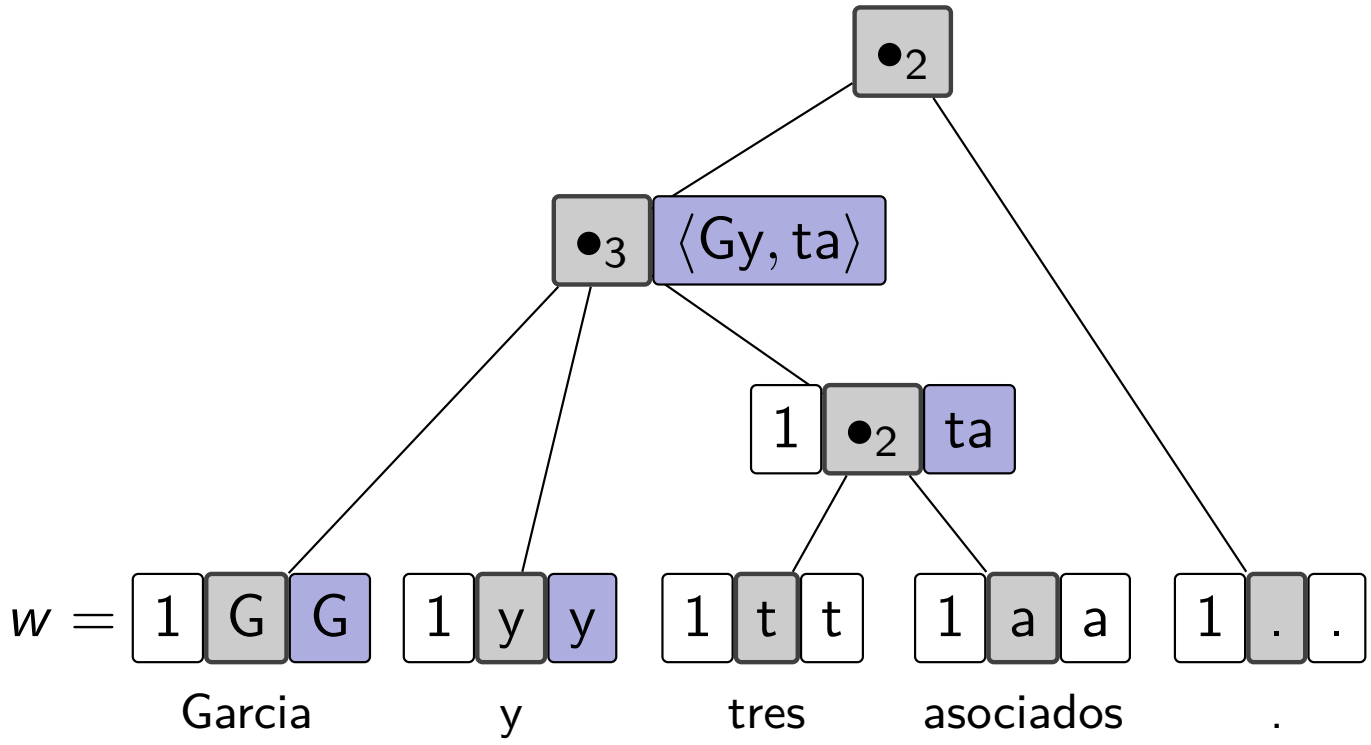

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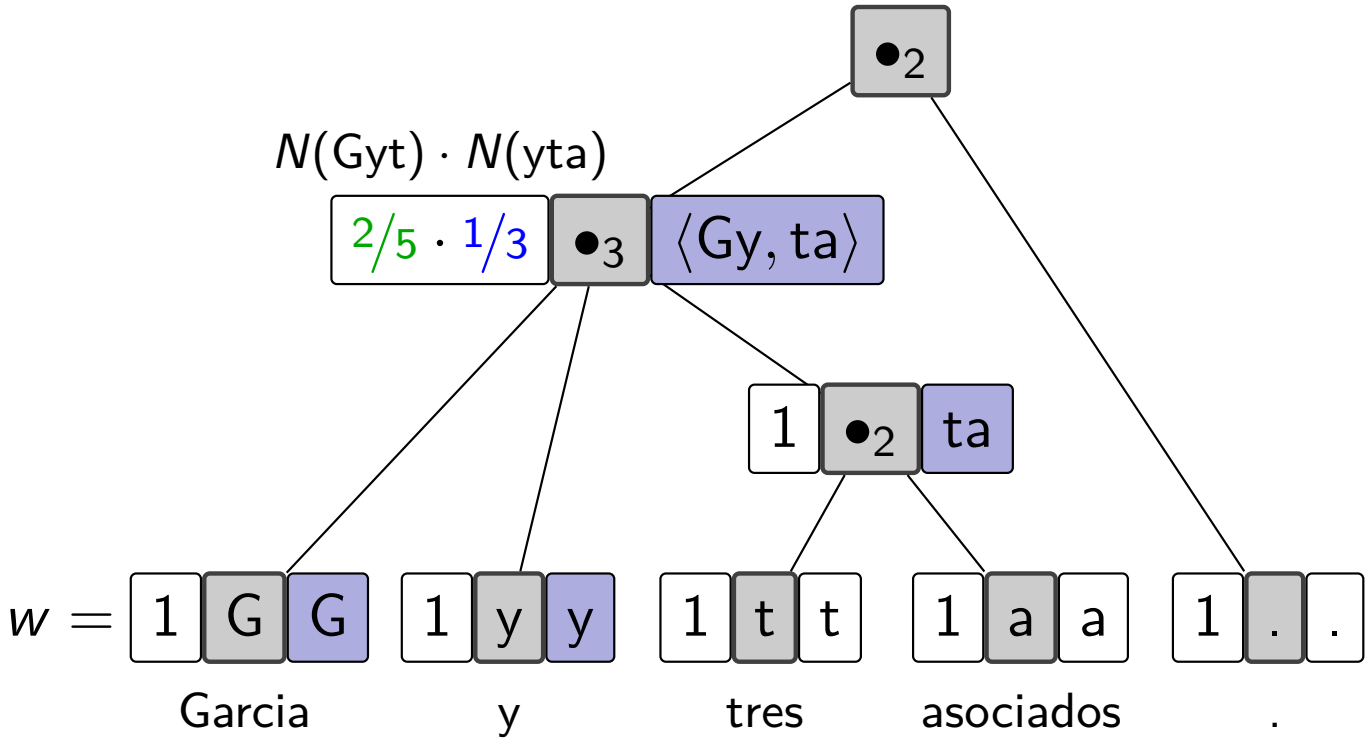

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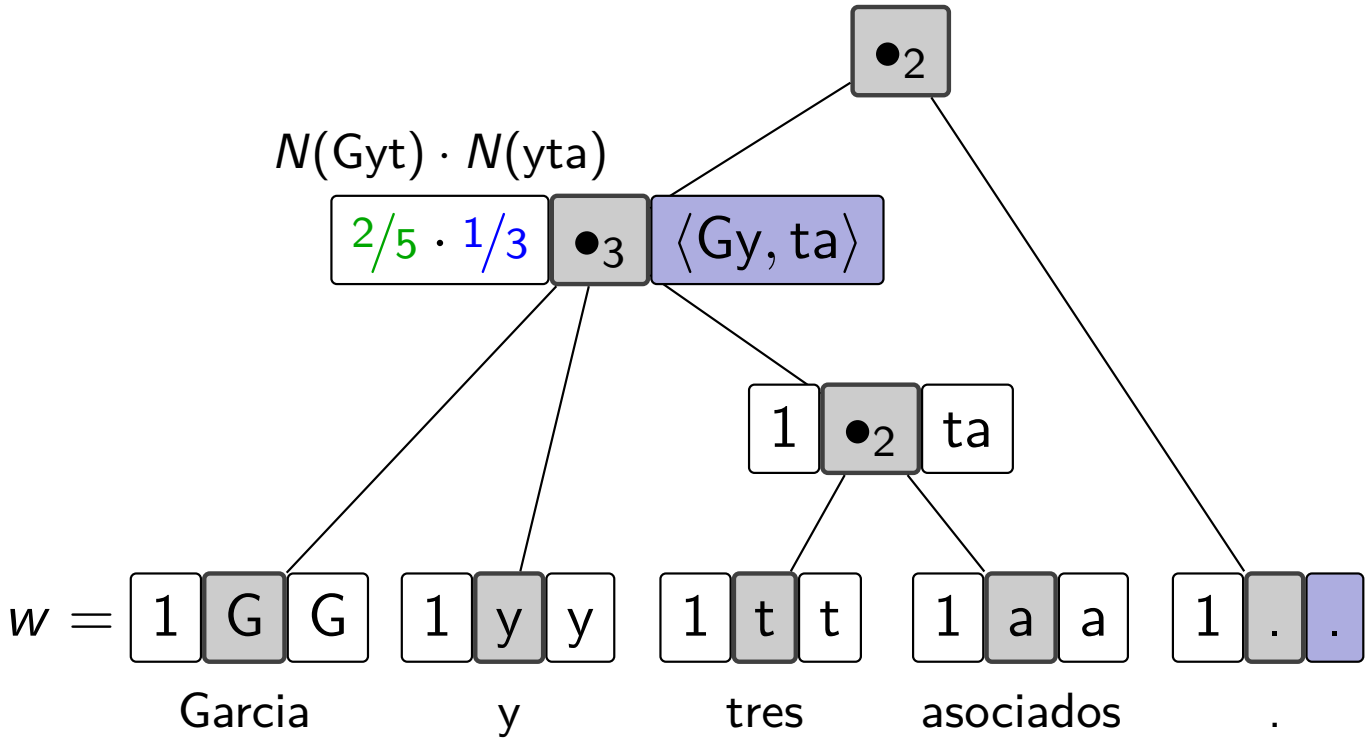

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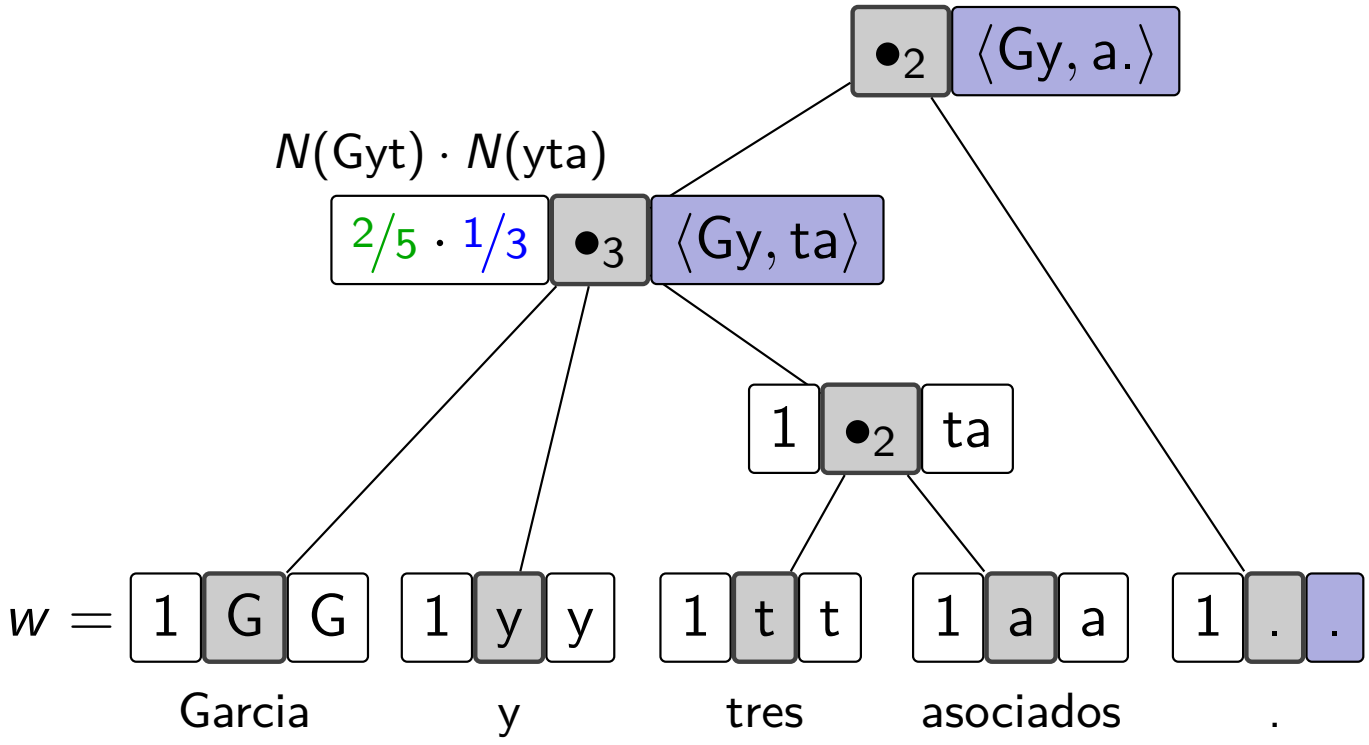

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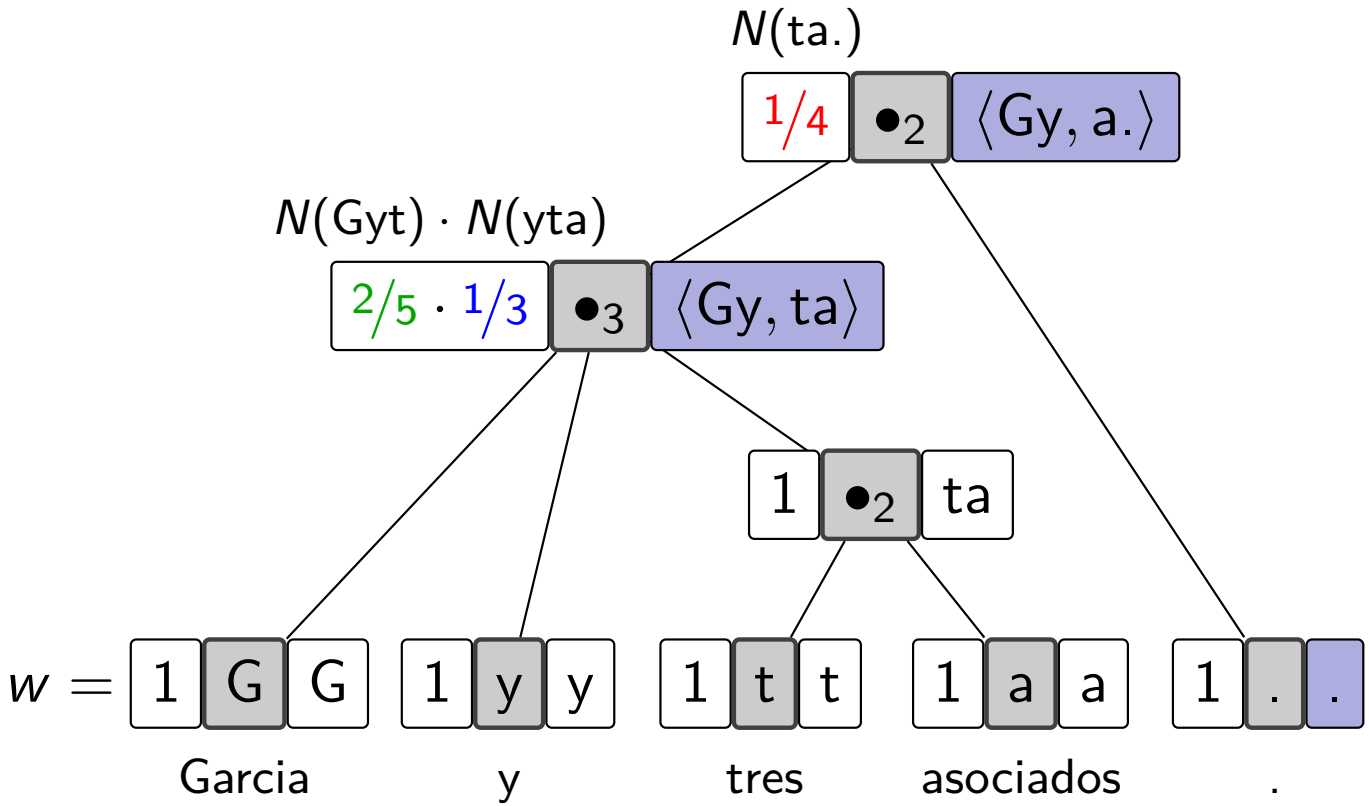

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$$h_{p_1,p_2}(f) = \text{argmax}_{e \in E} \underbrace{p_1}_{\text{by IRTG}}(f \mid e) \cdot \underbrace{p_2}_{\text{by } n\text{-gram model}}(e)$$

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References

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