Approximation of weighted automata with storage

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Language models in natural language processing

- regular grammars
- context-free grammars
- head grammars[1]
- multiple context-free grammars[2]

[1] or yields of tree adjoining grammars
[2] or linear context-free rewriting systems

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parsing complexities:

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- regular grammars $\mathcal{O}(n)$
  ≡ finite state automata
- context-free grammars (binarised) $\mathcal{O}(n^3)$
  ≡ pushdown automata [1, 2]
- head grammars[1] $\mathcal{O}(n^6)$
  ≡ embedded pushdown automata [3]
- multiple context-free grammars[2] (binarised, fan-out k) $\mathcal{O}(n^{3k})$
  ≡ k-restricted tree-stack automata [4]

problems:

- high parsing complexities → use *approximation*
- natural languages are ambiguous → use *weights* (semirings)
- lots of different models → use *automata with storage*

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Outline

1. Weighted automata with storage
2. Approximation of storage
3. Approximation of weighted automata with storage
4. Coarse-to-fine n-best parsing

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Storage

$S = (C, P, R, c_\mathsf{i})$

Example: Count

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every $r \in R$ finitely non-det.

($\forall c \in C\colon r(c)$ is finite)

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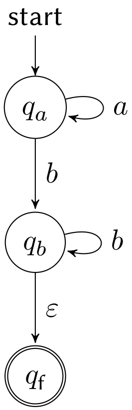
instances:
- Goldstine's [5] data storage: $P = \{C\}$
- Engelfriet's [6, 7] storage types: $R \subseteq C \dashrightarrow C$

ℕ-weighted automaton with Count-storage

automaton $\mathcal{M}$:



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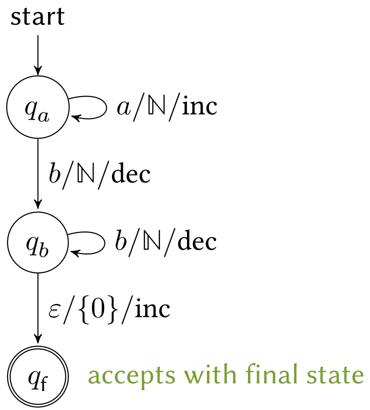

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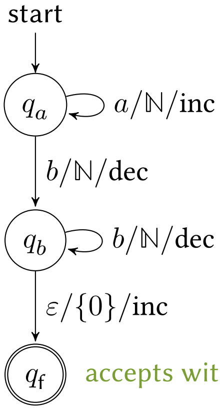

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$$L(\mathcal{M}) = \{a^n b^n \mid n \geq 1\}$$

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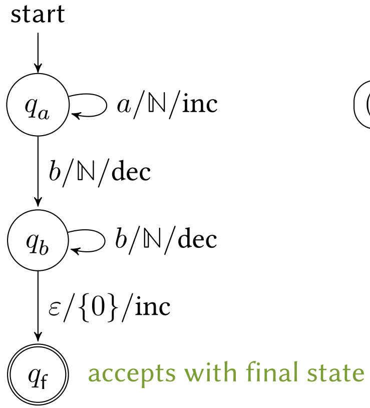
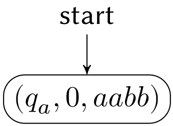


run of $\mathcal{M}$ on $aabb$:



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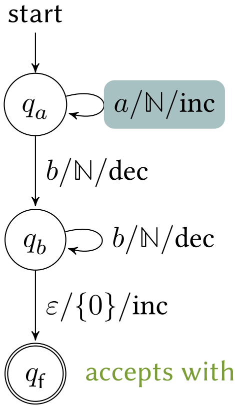

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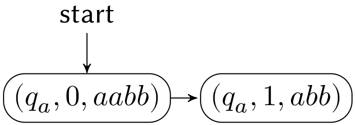

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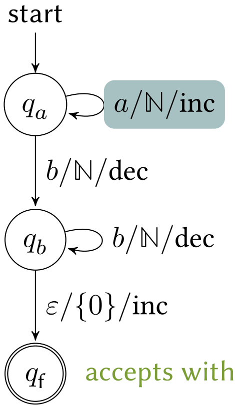

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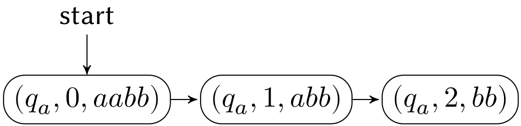

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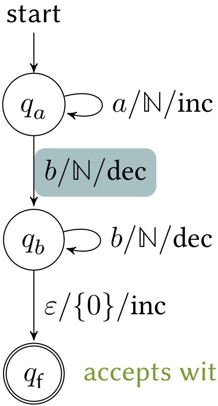

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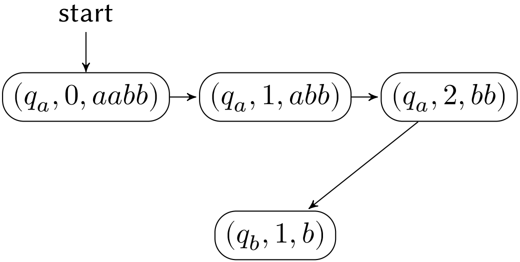

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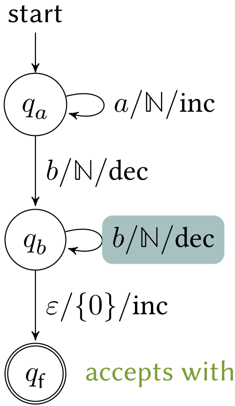

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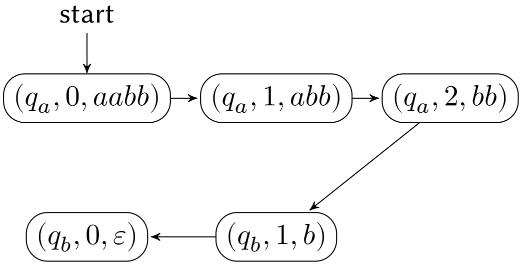

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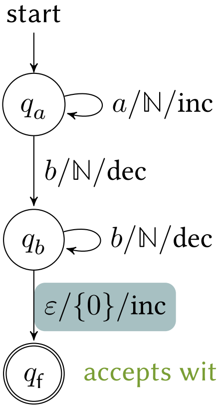

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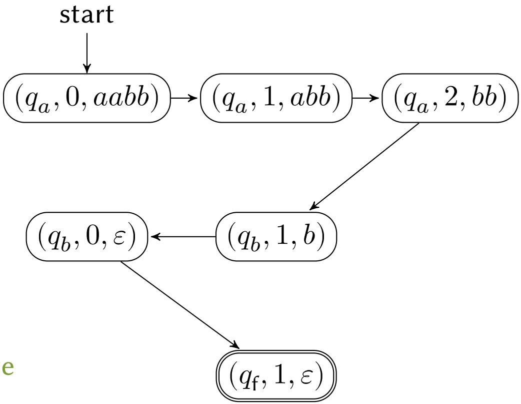

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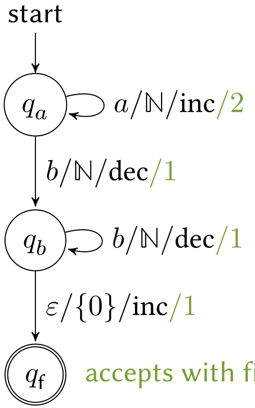

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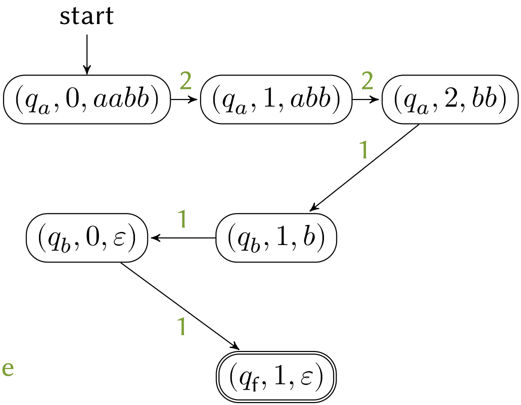
run of $\mathcal{M}$ on $aabb$: $(\mathbb{N}, +, \cdot, 0, 1)$



$L(\mathcal{M}) = \{a^n b^n \mid n \geq 1\}$ $\quad aabb \in L(\mathcal{M})$ $\quad [\![\mathcal{M}]\!](aabb) = 4$

$[\![\mathcal{M}]\!](a^n b^n) = 2^n$

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given: $S = (C, P, R, c_\mathsf{i})$, *approximation strategy* $A\colon C \dashrightarrow C'$

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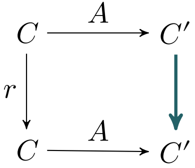

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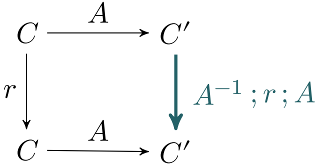

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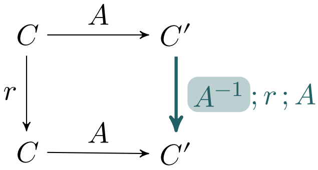

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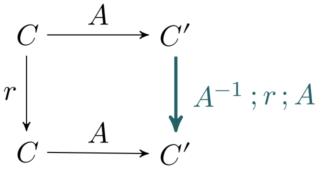

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$A(S)$ is only defined if $A^{-1}\,;r\,;A$ is finitely non-det. for each r.

An approximation of Count

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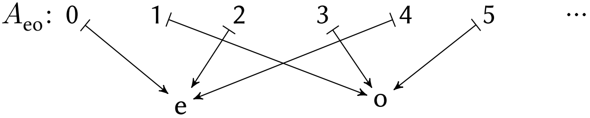

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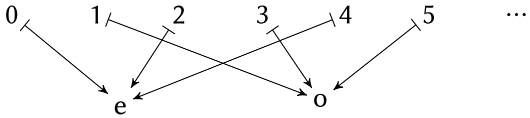

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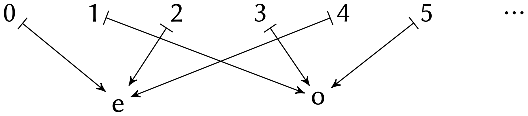

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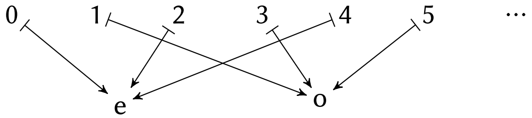

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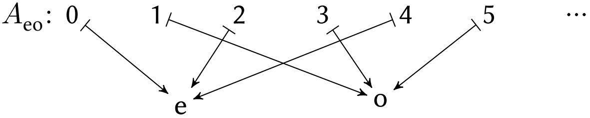

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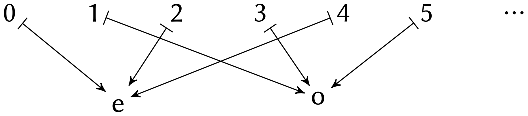

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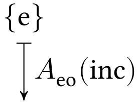

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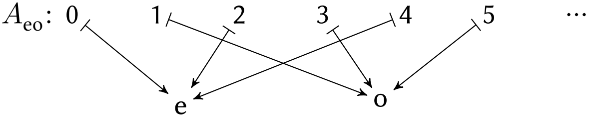

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$$\{0, 2, 4, \dots\} \xmapsto{A_{\text{eo}}} \{\text{e}\}$$

$$\{\text{e}\} \ \big\downarrow A_{\text{eo}}(\text{inc})$$

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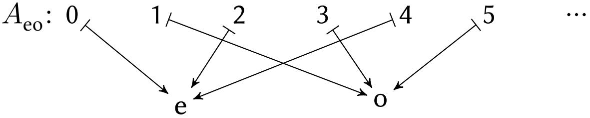

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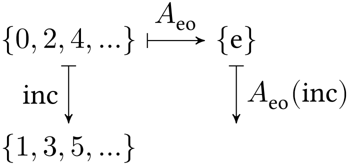
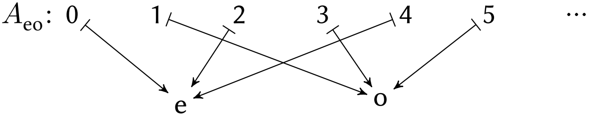


$A_{\text{eo}}(\text{Count}) = (\{\text{e}, \text{o}\},\ \{\{\text{e}, \text{o}\}, \{\text{e}\}\}, \qquad\qquad ,\ \text{e})$

An approximation of Count

$\text{Count} = (\mathbb{N},\ \ \{\mathbb{N}, \{0\}, \mathbb{N} \setminus \{0\}\},\ \ \{\text{inc}, \text{dec}\},\ \ 0)$



- $A_{\text{eo}}(\mathbb{N}) = \{\text{e}, \text{o}\} = A_{\text{eo}}(\mathbb{N} \setminus \{0\})$ $\qquad A_{\text{eo}}(\{0\}) = \{\text{e}\}$
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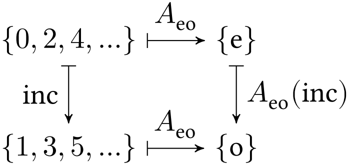

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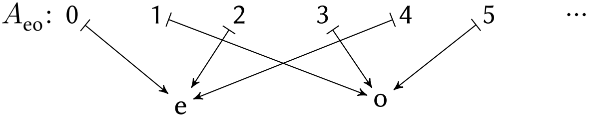

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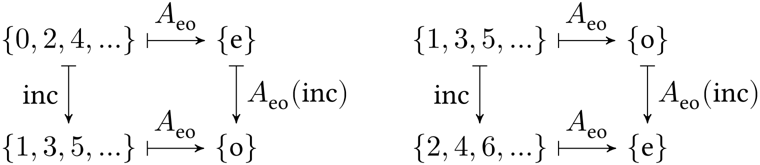

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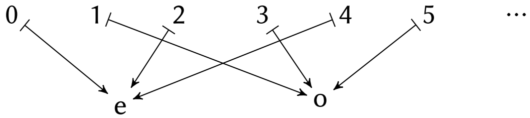

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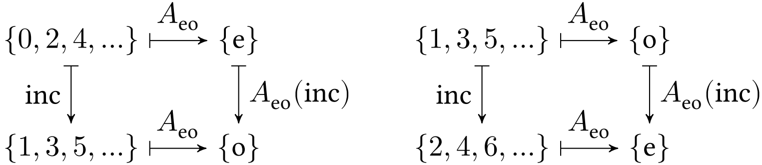

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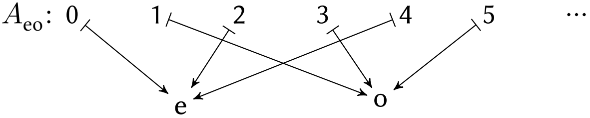

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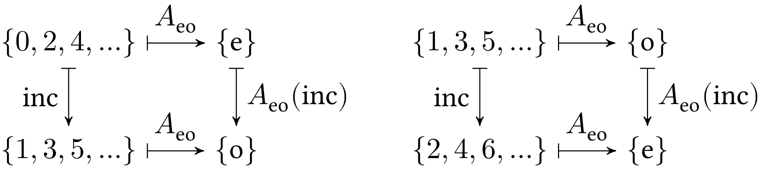

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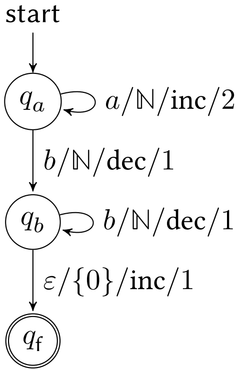
Outline

1. Weighted automata with storage
2. Approximation of storage
3. Approximation of weighted automata with storage
4. Coarse-to-fine n-best parsing

Approximation of $\mathbb{N}$-weighted aut. with Count-storage

automaton $\mathcal{M}$:



$$L(\mathcal{M}) = \{a^n b^n \mid n \geq 1\}$$

$$[\![\mathcal{M}]\!](a^n b^n) = 2^n$$

Approximation of ℕ-weighted aut. with Count-storage

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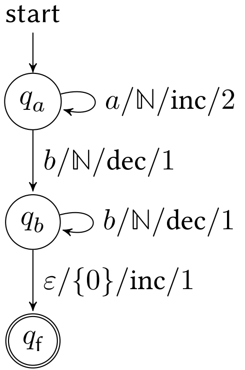
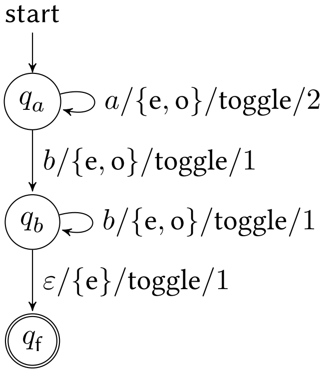


automaton $A_{\mathrm{eo}}(\mathcal{M})$:



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Approximation of $\mathbb{N}$-weighted aut. with Count-storage

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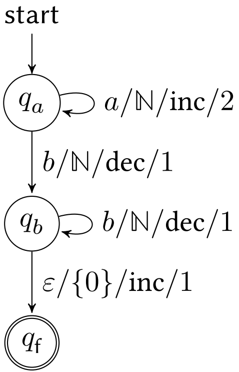

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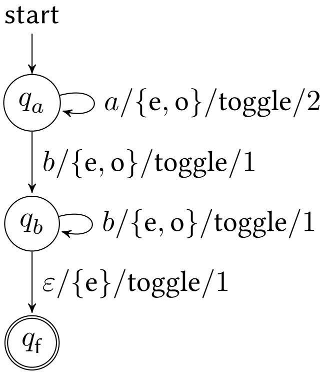

$$L(A_{\text{eo}}(\mathcal{M})) = \{a^m b^n \mid m \geq 0, n \geq 1, m \equiv n \mod 2\}$$

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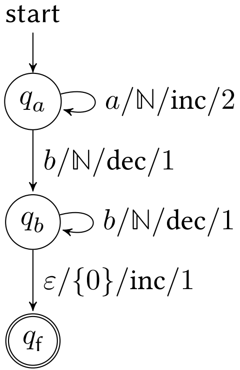

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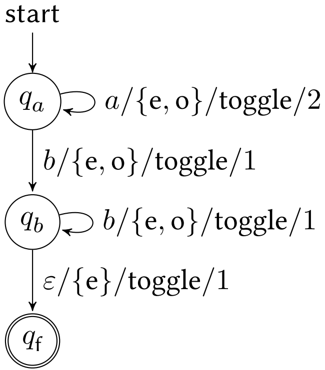

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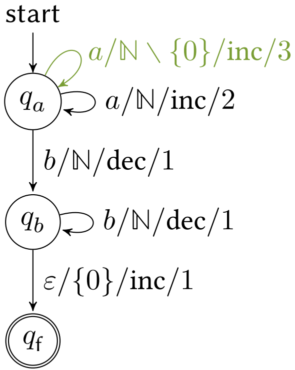

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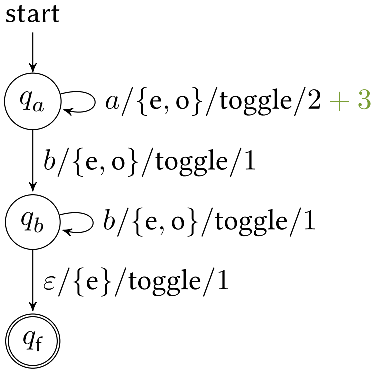

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$$[\![A_{\mathrm{eo}}(\mathcal{M})]\!](a^m b^n) = (2+3)^m \quad m \equiv n \mod 2$$

Approximation of weighted automata with storage

Theorem (unweighted).

Let $\mathcal{M}$ be an (S, Σ)-automaton and A be an S-proper approximation strategy.

- If A is *total*, then $L(A(\mathcal{M})) \supseteq L(\mathcal{M})$.
- If A is *injective*, then $L(A(\mathcal{M})) \subseteq L(\mathcal{M})$.

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Theorem (weighted).

Let $\mathcal{M}$ be an (S, Σ, K)-automaton, A be an S-proper approximation strategy, $\leq$ be a partial order on K, and K be positively $\leq$-ordered.

- If A is *total*, then $[\![A(\mathcal{M})]\!](w) \geq [\![\mathcal{M}]\!](w)$ for every $w \in \Sigma^*$.
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Theorem (unweighted) is a corollary of Theorem (weighted)

[by setting $K = \mathbb{B}$]

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1. Weighted automata with storage
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3. Approximation of weighted automata with storage
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n-best parsing (for automata)

parsing

Input: an automaton $\mathcal{M}$, a word w

Output: the set of all runs of $\mathcal{M}$ on w

n-best parsing (for automata)

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n-best parsing

Input: a K-weighted automaton $\mathcal{M}$, a word w, a number n

Output: a sequence of n best (w.r.t. weight) runs of $\mathcal{M}$ on w

Coarse-to-fine n-best parsing

Idea: 1. recognise word with a *superset* approximation $A(\mathcal{M})$

2. use runs of $A(\mathcal{M})$ to **reduce search space** for runs of $\mathcal{M}$

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Input: a K-weighted automaton $\mathcal{M}$, a word w, a number n, proper *total* approximation strategy A

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$$(\mathcal{M}, w) \xrightarrow{\text{2-best parse}}$$

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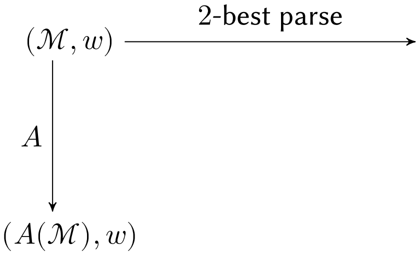

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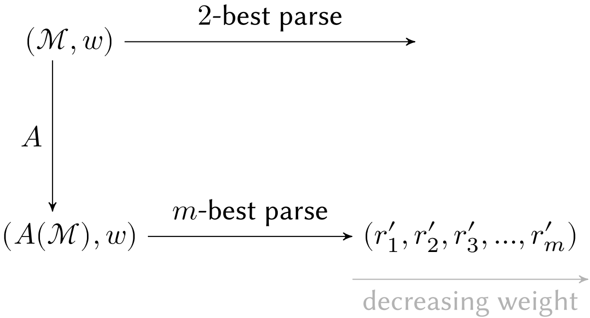

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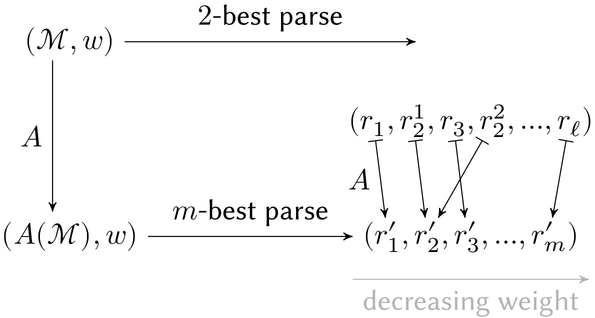

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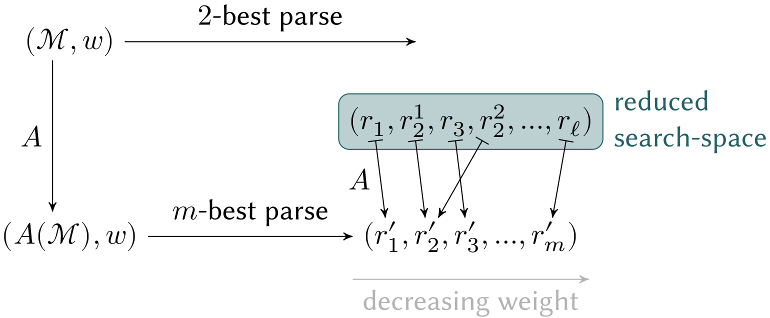

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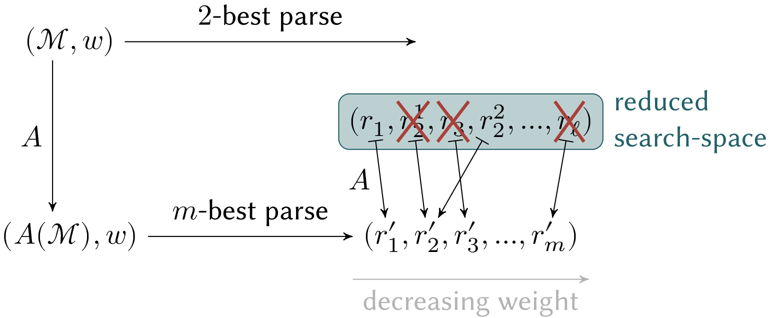

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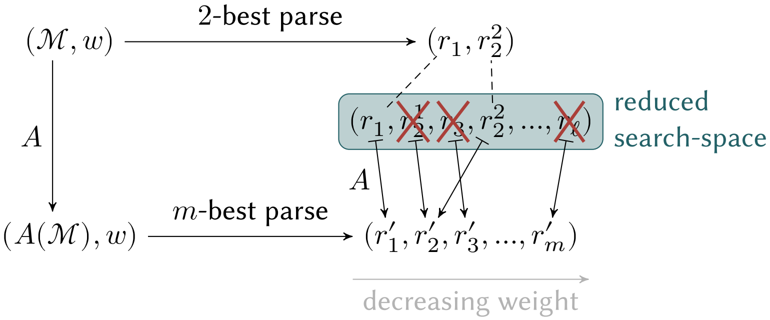

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Thank you for your attention.

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