Chomsky-Schützenberger Characterisation for Multiple Context-Free Languages

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Classical result

Theorem (Chomsky and Schützenberger 1963, Prop. 2)

For every CFL L there are

- a Dyck language D,
- a regular language R, and
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$$L = h(D \cap R) .$$

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$$L\colon \mathbf{A} \xrightarrow{r_1} B\ C\ , \quad B \xrightarrow{r_2} \alpha, \quad C \xrightarrow{r_3} \beta$$

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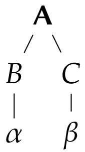

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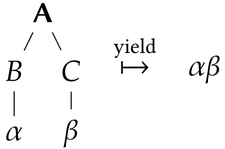

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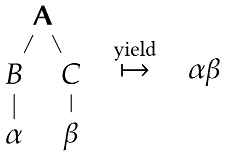

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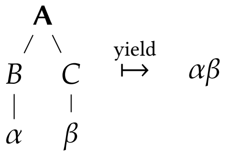

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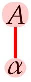

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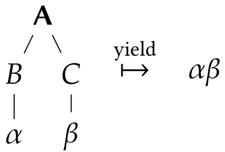

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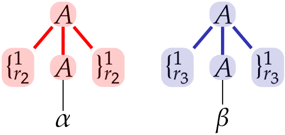

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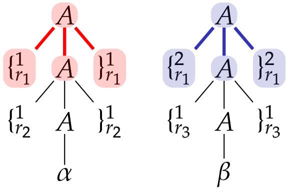

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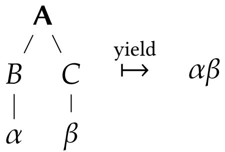

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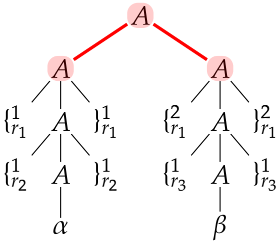

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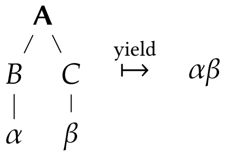

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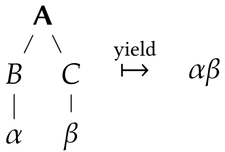

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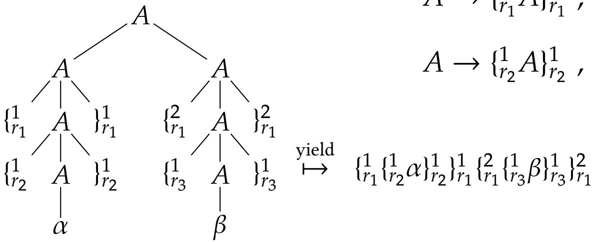

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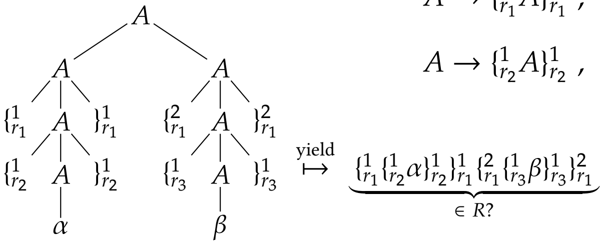

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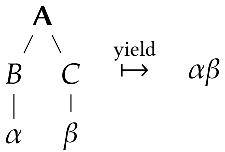

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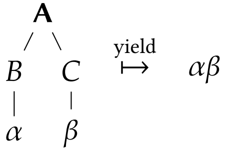

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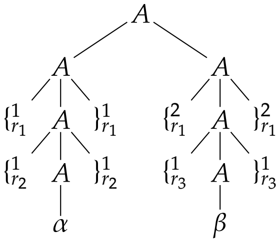

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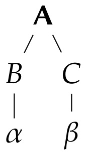

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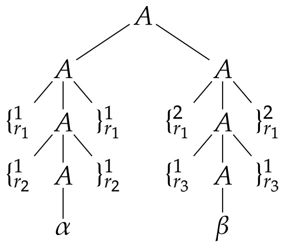

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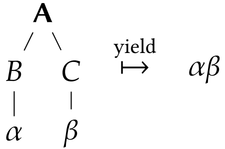

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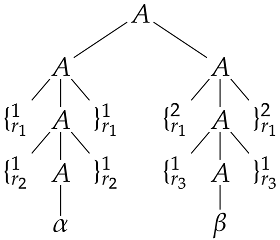

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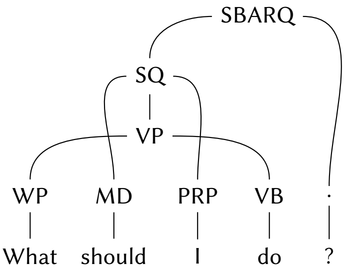
Non-projective trees

Example (Evang and Kallmeyer 2011)



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Non-projective trees

gaps / crossing edges

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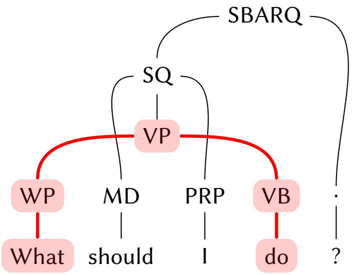

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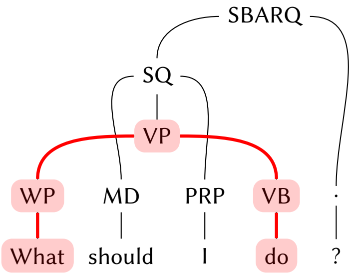

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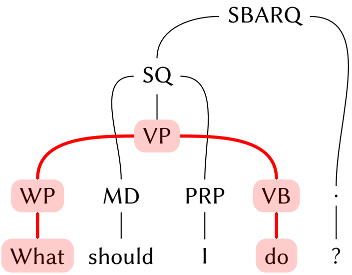

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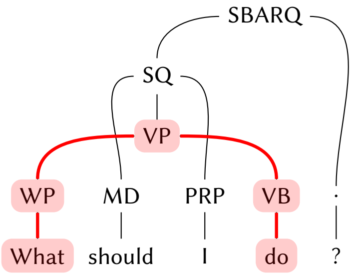

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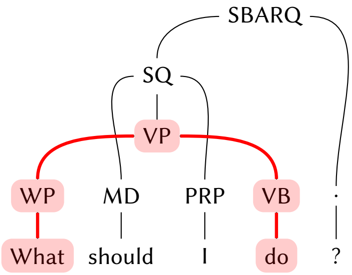

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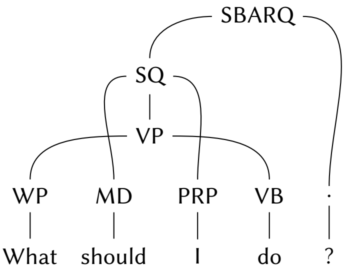

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$$\left[\pi_1^1 \pi_2^1, \beta\pi_2^2\right]\colon (\Sigma^*)^1 \times (\Sigma^*)^2 \to (\Sigma^*)^2$$

$$\left[\pi_1^1 \pi_2^1, \beta\pi_2^2\right]\Big((\underbrace{\alpha\gamma}_{\pi_1^1}), (\underbrace{\alpha}_{\pi_2^1}, \underbrace{\beta}_{\pi_2^2}) \Big) = (\alpha\gamma\alpha, \beta\beta)$$

Word-tuple functions

a k-ary word-tuple function

$$f\colon (\Sigma^*)^{m_1} \times \cdots \times (\Sigma^*)^{m_k} \to (\Sigma^*)^m$$

Example

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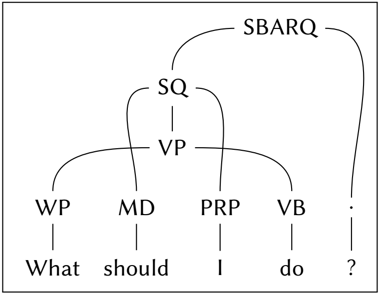
For MCFG: only *linear* word-tuple functions

does not copy argument components

Multiple context-free grammars (MCFGs)



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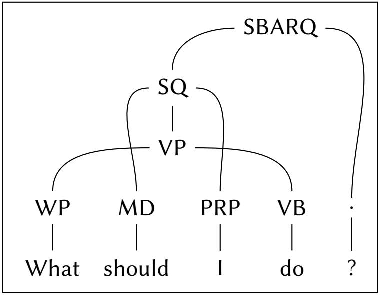

- gaps $\rightsquigarrow$ commas

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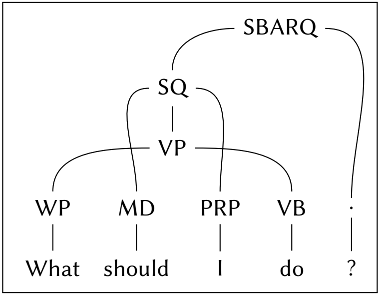

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MCFG generating the tree:

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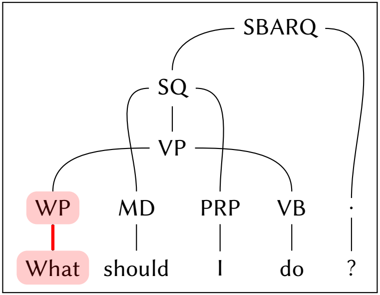

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MCFG generating the tree:

WP → [What]

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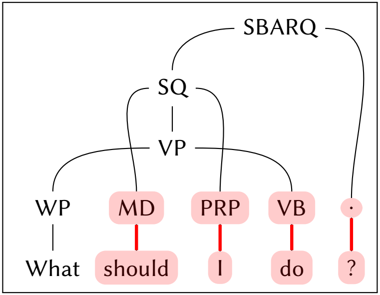

MCFG generating the tree:

WP → [What]

MD → [should]

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. → [?]

- gaps ⇝ commas

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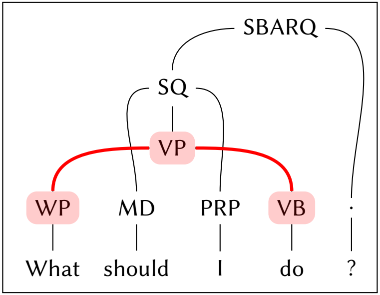

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MCFG generating the tree:

$$\mathrm{WP} \rightarrow [\mathrm{What}]$$
$$\mathrm{MD} \rightarrow [\mathrm{should}]$$
$$\mathrm{PRP} \rightarrow [\mathrm{I}]$$
$$\mathrm{VB} \rightarrow [\mathrm{do}]$$
$$. \rightarrow [?]$$
$$\mathrm{VP} \rightarrow [\pi_1^1, \pi_2^1](\mathrm{WP}, \mathrm{VB})$$

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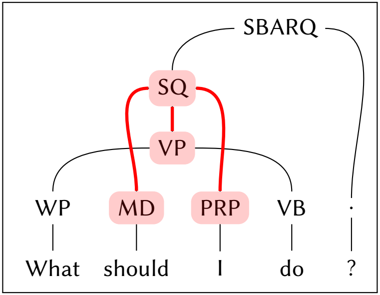

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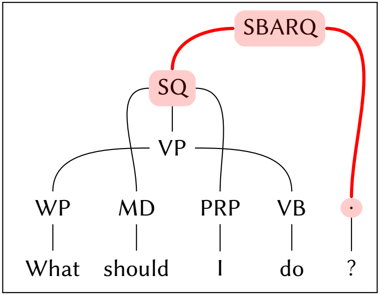

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MCFG generating the tree:

$$\begin{aligned}
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\mathrm{SQ} &\rightarrow [\pi_2^1 \pi_1^1 \pi_3^1 \pi_2^2](\mathrm{MD}, \mathrm{VP}, \mathrm{PRP}) \\
\mathbf{SBARQ} &\rightarrow [\pi_1^1 \pi_2^1](\mathrm{SQ}, .)
\end{aligned}$$

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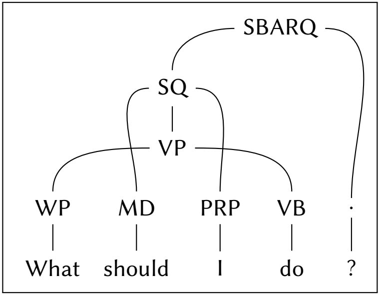

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MCFG generating the tree:

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Chomsky-Schützenberger for MCFG

Theorem

For every multiple context-free language $\mathscr{L}$ there are

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$$\mathscr{L} = h(\mathscr{D} \cap R) .$$

Bracket alphabet

Two types of brackets: rule $r\colon A \to [u_1, \ldots, u_m](A_1, \ldots, A_k)$

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$\rightsquigarrow h(w)$ is derived in the j-th symbol of u_i

Multiple Dyck language

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- given by an MCFG

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- similar to classical *Dyck language*

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Dyck language

- $A \to \sigma$
- $A \to AA$
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Multiple Dyck derivation

$$R\colon \mathbf{S} \xrightarrow{r_1} [\pi^1_1\pi^1_2\pi^2_1](A,B), \quad A \xrightarrow{r_2} [\pi^1_1,\pi^1_2](C,C), \quad B \xrightarrow{r_3} [\alpha], \quad C \xrightarrow{r_4} [\beta]$$

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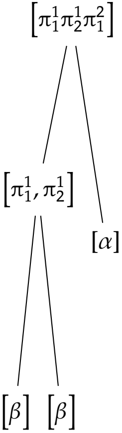

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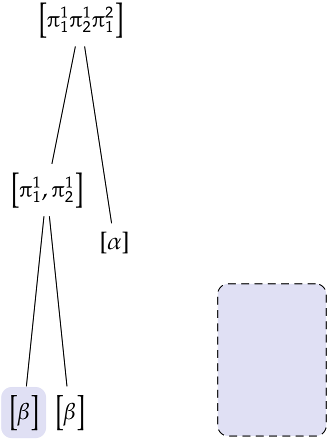

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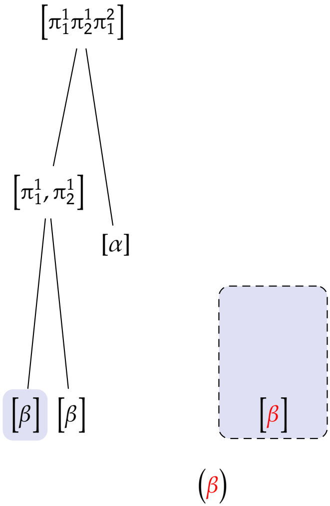

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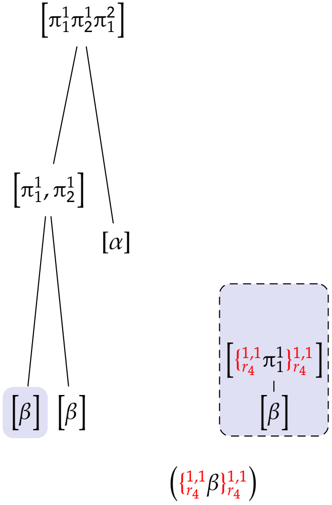

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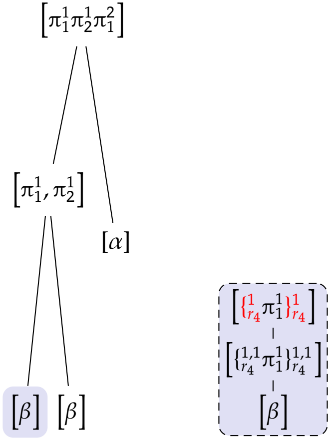

$$\left(\{_{r_4}^{1}\{_{r_4}^{1,1}\beta\}_{r_4}^{1,1}\}_{r_4}^{1}\right)$$

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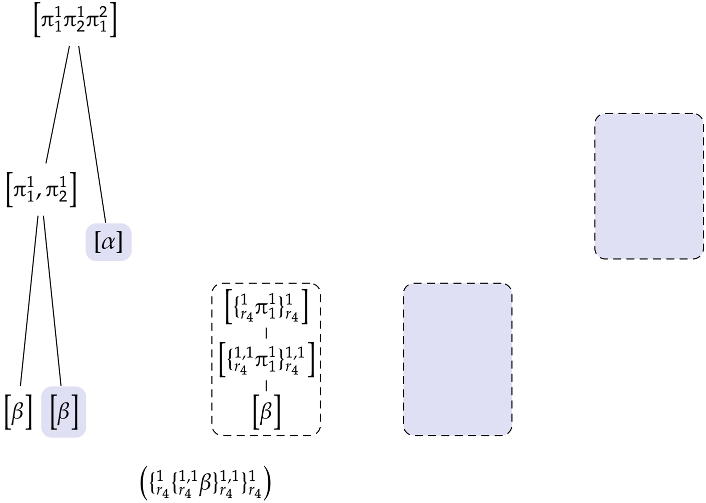

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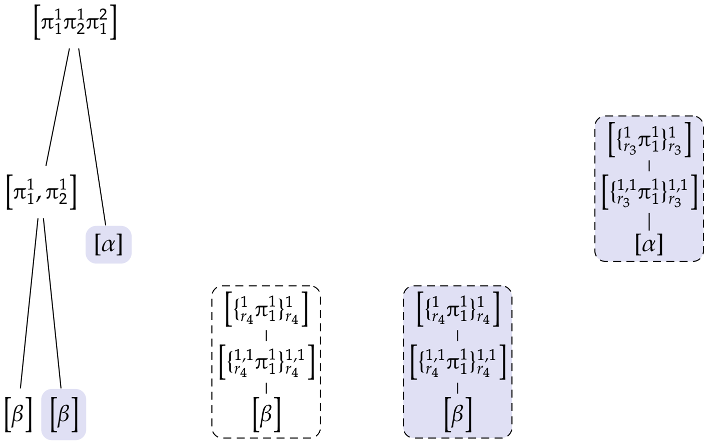

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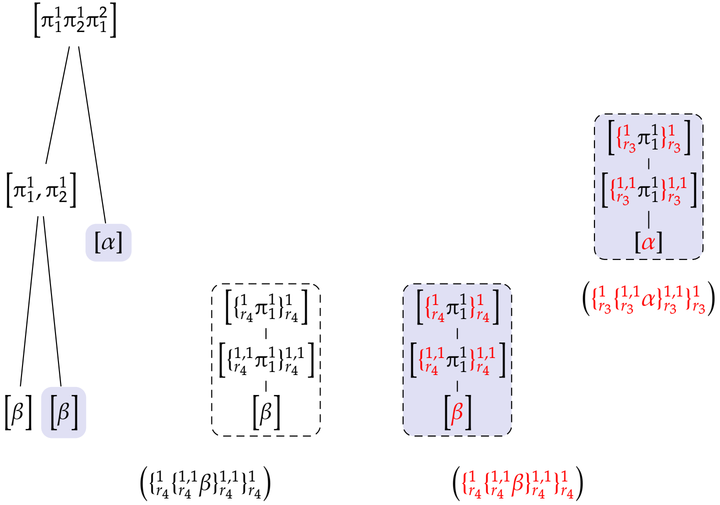

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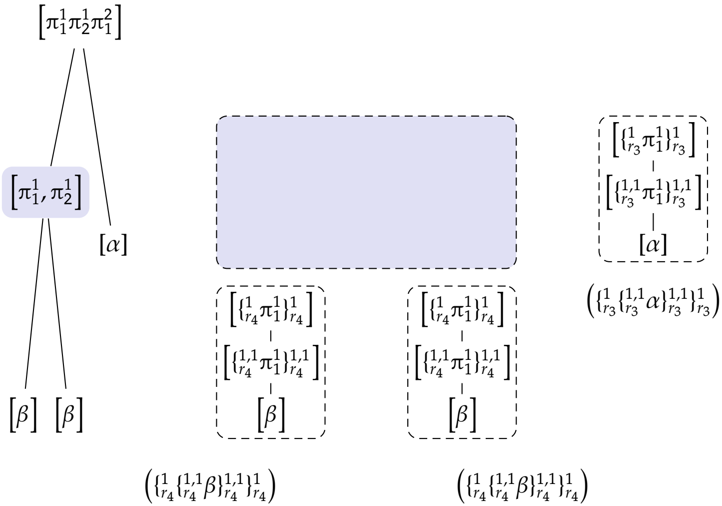

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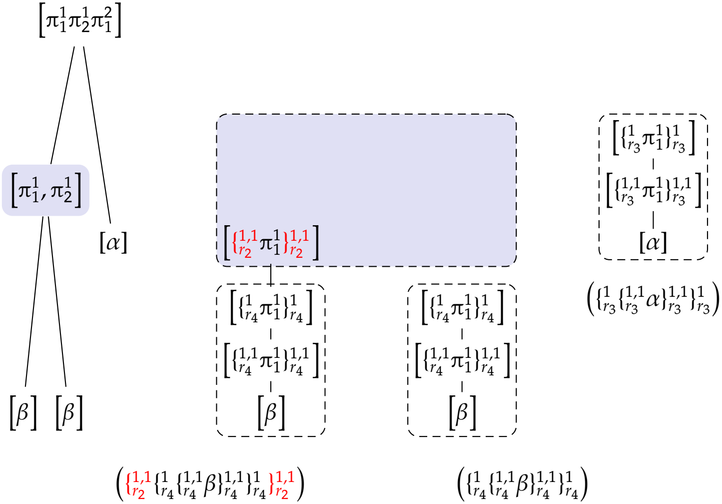

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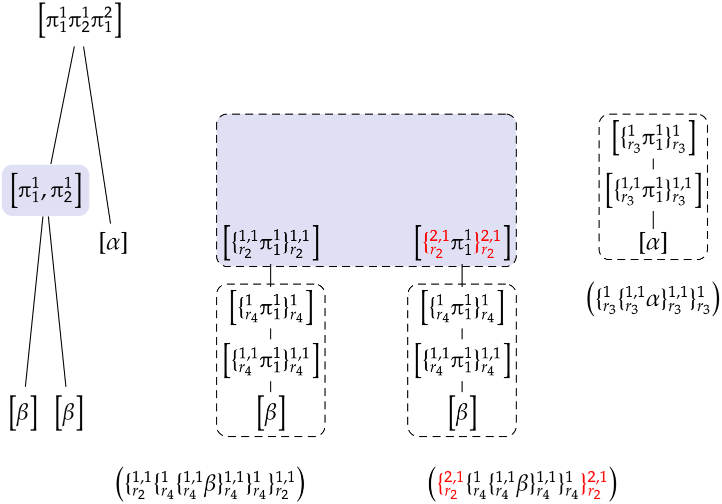

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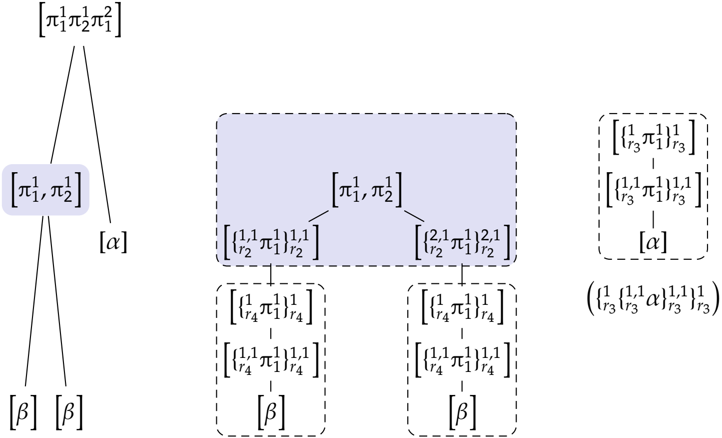

$$\left(\{^{1,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{1,1}_{r_2} \quad , \quad \{^{2,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{2,1}_{r_2}\right)$$

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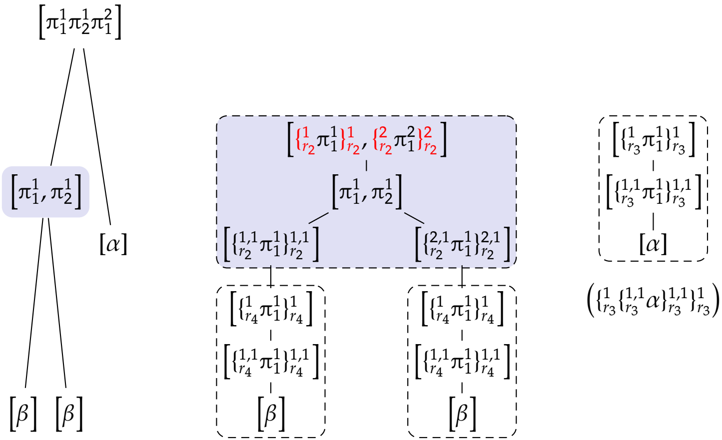

$$\left(\{^{1,1}_{r_3}\{^{1,1}_{r_3}\alpha\}^{1,1}_{r_3}\}^{1}_{r_3}\right)$$

$$\left(\{^{1}_{r_2}\{^{1,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{1,1}_{r_2}\}^{1}_{r_2} \quad , \quad \{^{2}_{r_2}\{^{2,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{2,1}_{r_2}\}^{2}_{r_2}\right)$$

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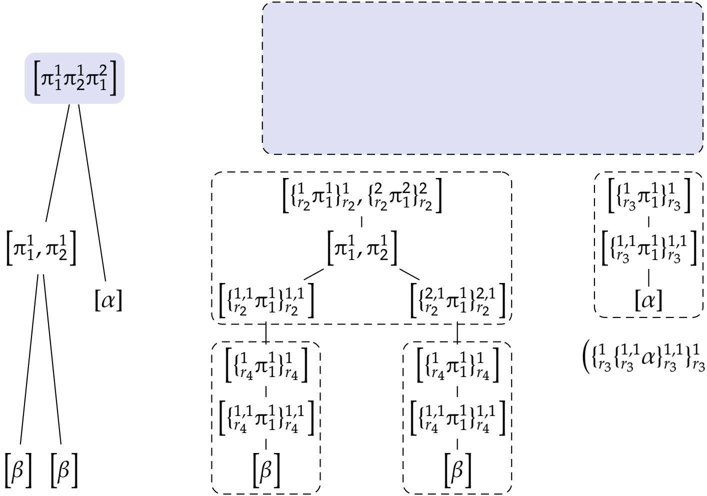

$$\left(\{_{r_3}^1\{_{r_3}^{1,1}\alpha\}_{r_3}^{1,1}\}_{r_3}^1\right)$$

$$\left(\{_{r_2}^1\{_{r_2}^{1,1}\{_{r_4}^1\{_{r_4}^{1,1}\beta\}_{r_4}^{1,1}\}_{r_4}^1\}_{r_2}^{1,1}\}_{r_2}^1 \quad , \quad \{_{r_2}^2\{_{r_2}^{2,1}\{_{r_4}^1\{_{r_4}^{1,1}\beta\}_{r_4}^{1,1}\}_{r_4}^1\}_{r_2}^{2,1}\}_{r_2}^2\right)$$

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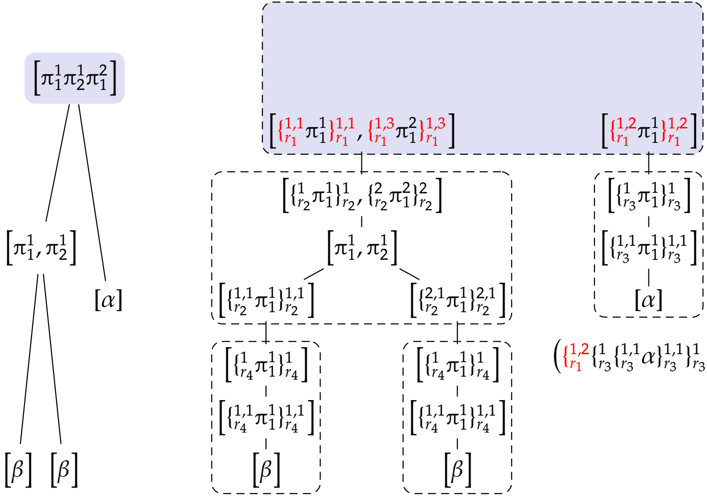

$$\left(\{^{1,2}_{r_1}\{^{1}_{r_3}\{^{1,1}_{r_3}\alpha\}^{1,1}_{r_3}\}^{1}_{r_3}\}^{1,2}_{r_1}\right)$$

$$\left(\{^{1,1}_{r_1}\{^{1}_{r_2}\{^{1,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{1,1}_{r_2}\}^{1}_{r_2}\}^{1,1}_{r_1}\ ,\ \{^{1,3}_{r_1}\{^{2}_{r_2}\{^{2,1}_{r_2}\{^{1}_{r_4}\{^{1,1}_{r_4}\beta\}^{1,1}_{r_4}\}^{1}_{r_4}\}^{2,1}_{r_2}\}^{2}_{r_2}\}^{1,3}_{r_1}\right)$$

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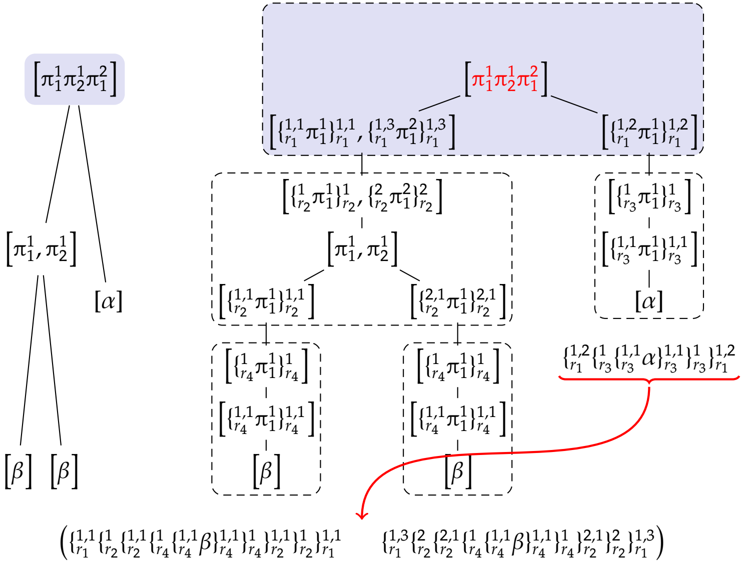
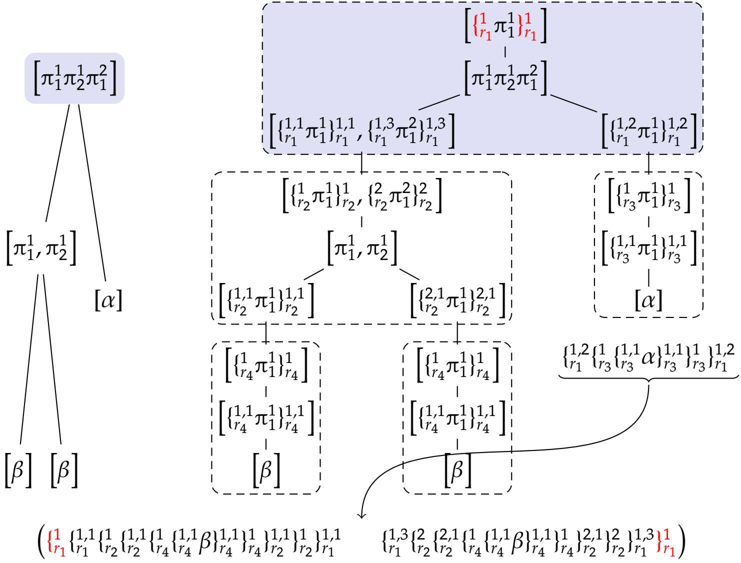


Multiple Dyck derivation

$$R\colon \mathbf{S} \xrightarrow{r_1} [\pi_1^1\pi_2^1\pi_1^2](A,B),\quad A \xrightarrow{r_2} [\pi_1^1,\pi_2^1](C,C),\quad B \xrightarrow{r_3} [\alpha],\quad C \xrightarrow{r_4} [\beta]$$



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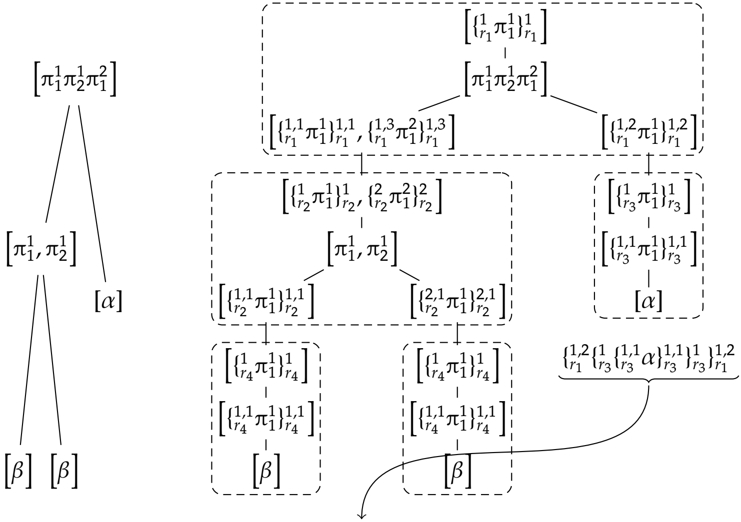

$$\left(\{^{1}_{r_1} \{^{1,1}_{r_1} \{^{1}_{r_2} \{^{1,1}_{r_2} \{^{1}_{r_4} \{^{1,1}_{r_4} \beta \}^{1,1}_{r_4} \}^{1}_{r_4} \}^{1,1}_{r_2} \}^{1}_{r_2} \}^{1,1}_{r_1} \quad \{^{1,3}_{r_1} \{^{2}_{r_2} \{^{2,1}_{r_2} \{^{1}_{r_4} \{^{1,1}_{r_4} \beta \}^{1,1}_{r_4} \}^{1}_{r_4} \}^{2,1}_{r_2} \}^{2}_{r_2} \}^{1,3}_{r_1} \}^{1}_{r_1} \right)$$

Chomsky-Schützenberger for weighted MCFL

Theorem

For every zero-sum and zero-product free semiring S and every S-weighted MCFL $\mathscr{L}^{\mathrm{w}}$ there are

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Idea

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2. use unweighted construction
3. original MCFG weights in R when $\{^1_r$ is read

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Outlook

- Define a *parsing algorithm* for MCFG based on their Chomsky-Schützenberger characterisation (similar to Hulden 2011).

References

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[3] Mans Hulden. "Parsing CFGs and PCFGs with a Chomsky-Schützenberger Representation". 2011.

[4] Wolfgang Maier and Anders Søgaard. "Treebanks and mild context-sensitivity". 2008.