

# A direct link between Tree-Adjoining and Context-Free Tree Grammars

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## Theorem (Kepser and Rogers, 2011)

Let  $\mathcal{L}$  be a tree language. The following are equivalent:

- ▶  $\mathcal{L}$  is definable by a **non-strict tree adjoining grammar**.
- ▶  $\mathcal{L}$  is definable by a **monadic linear context-free tree grammar**.

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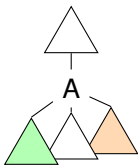
then it is also definable by a

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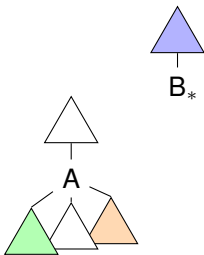
new **direct** construction:

productions **resemble shape** of elementary trees

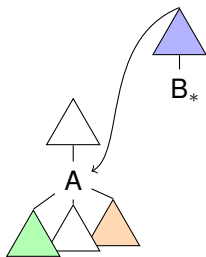
# (non-strict) tree adjoining grammars



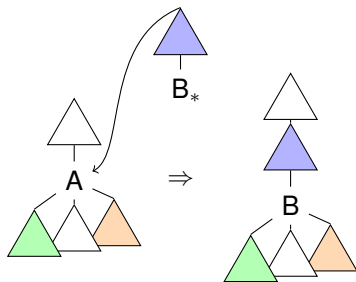
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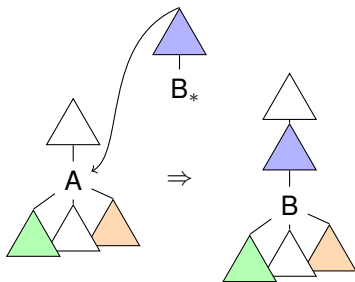
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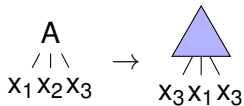
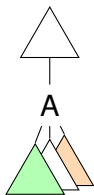
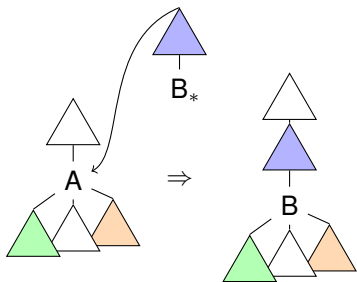
(non-strict) tree  
adjoining grammars

context-free tree grammars

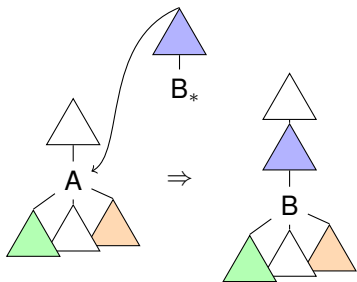


# (non-strict) tree adjoining grammars

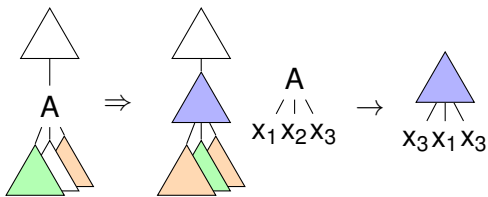
# context-free tree grammars



# (non-strict) tree adjoining grammars



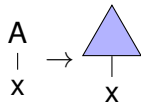
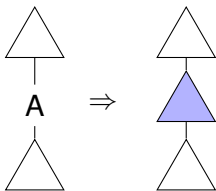
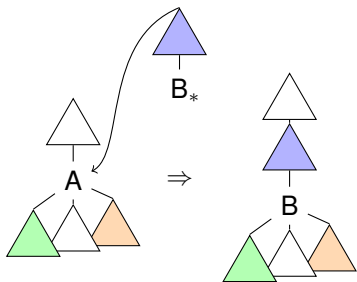
# context-free tree grammars



(non-strict) tree  
adjoining grammars

vs.

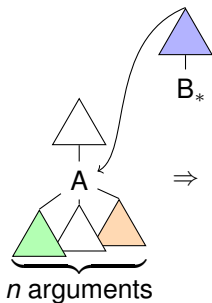
linear monadic  
context-free tree grammars



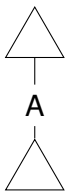
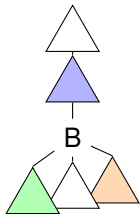
(non-strict) tree  
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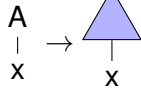
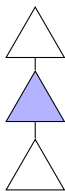
linear monadic  
context-free tree grammars



$\Rightarrow$



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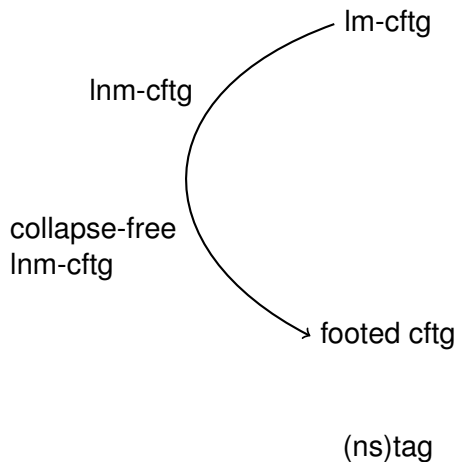


# equivalence proof by Kepser and Rogers

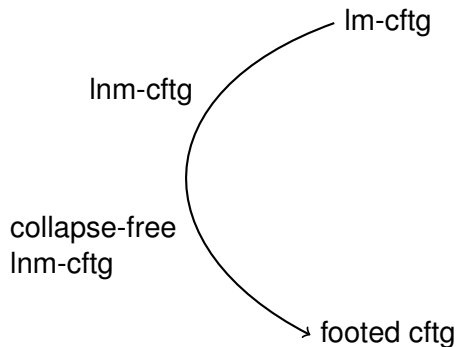
lm-cftg

(ns)tag

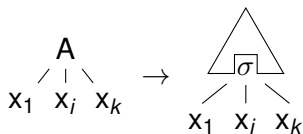
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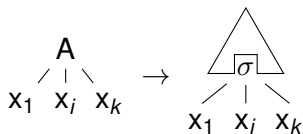
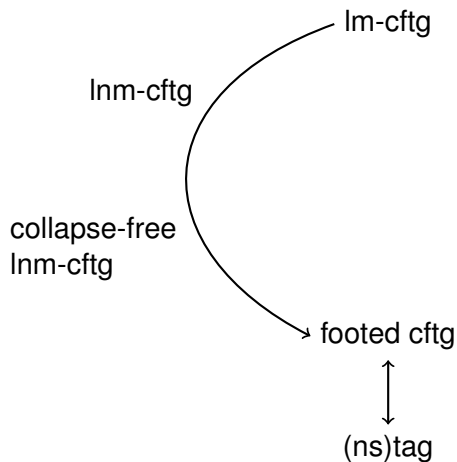


(ns)tag

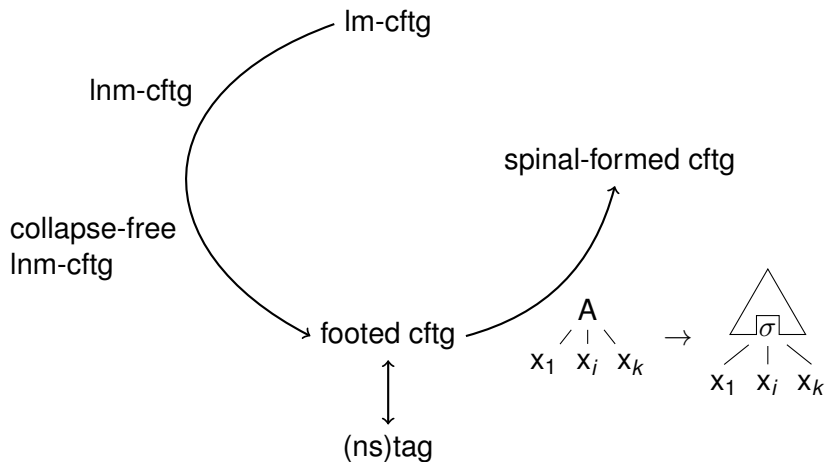




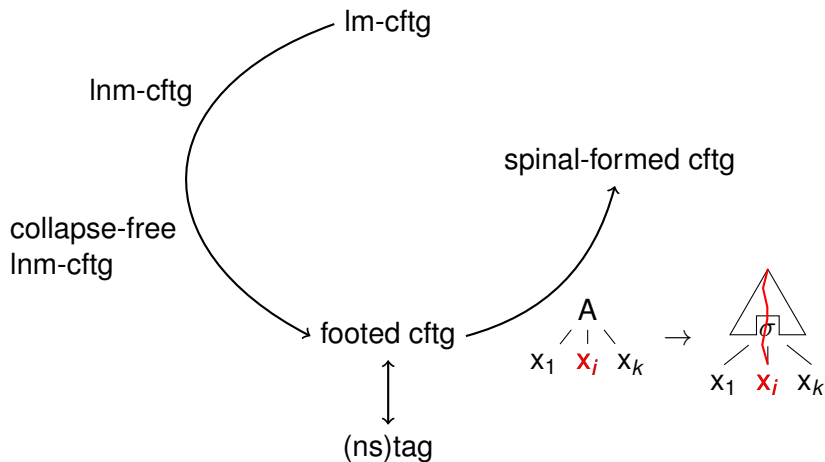
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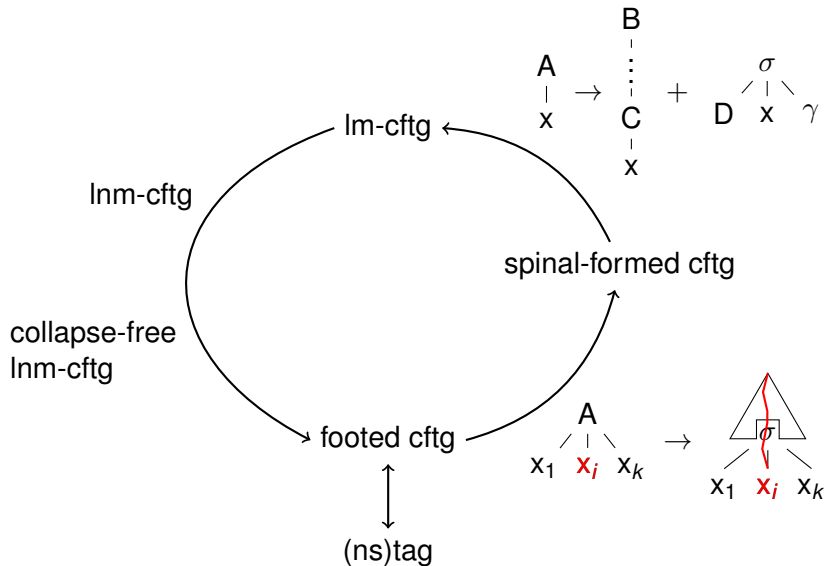
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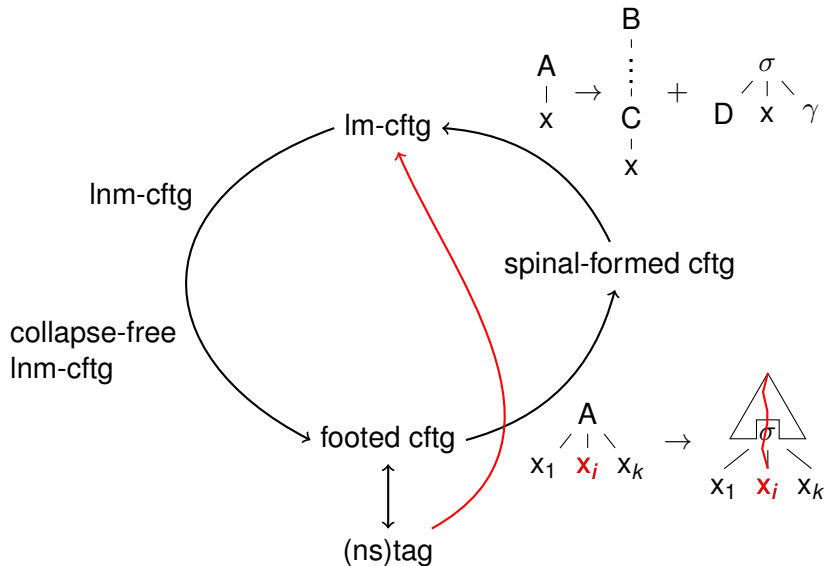
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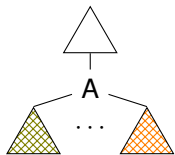


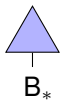
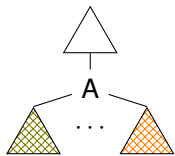
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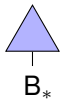
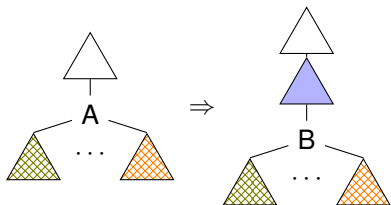


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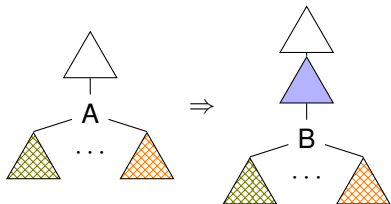




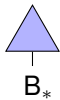


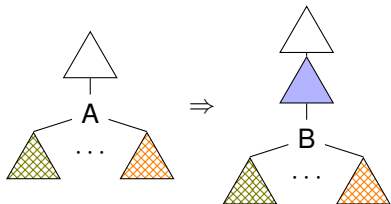






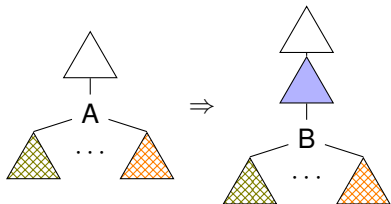
► order of siblings unaffected





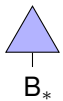
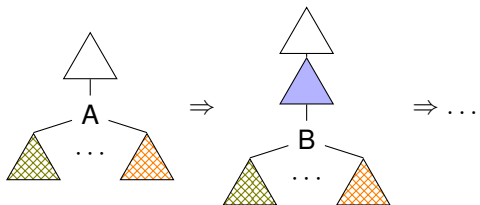
- ▶ order of siblings unaffected
- ▶ conceive siblings as 1 argument

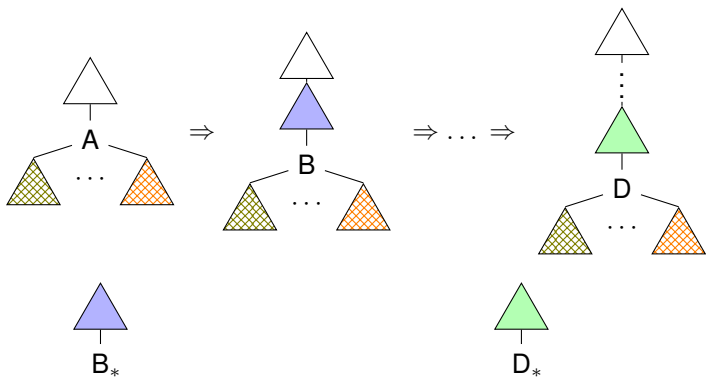


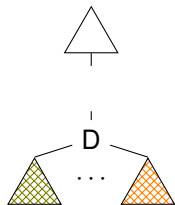
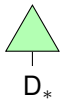
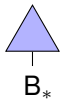
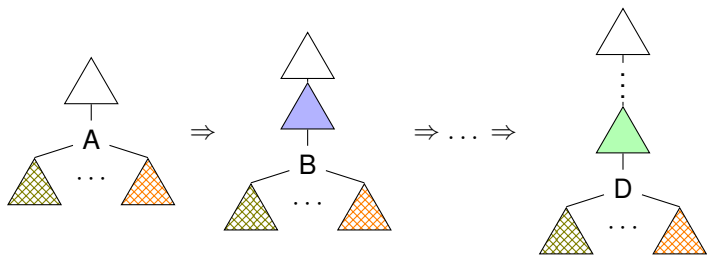


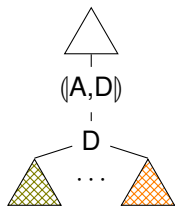
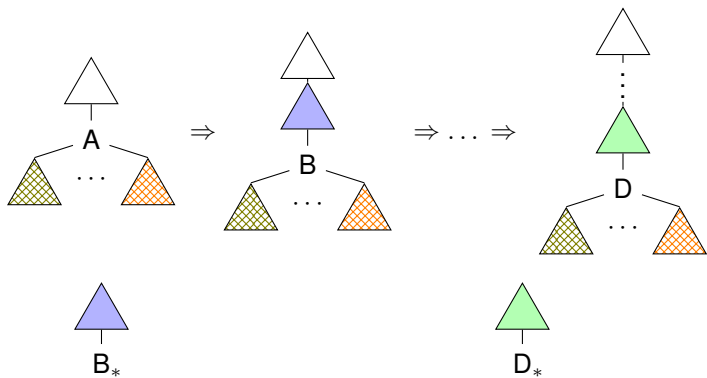
- ▶ order of siblings unaffected
- ▶ conceive siblings as 1 argument
- ▶ plug parent node on top

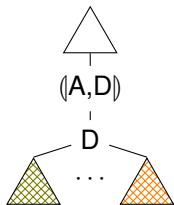
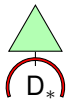
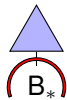
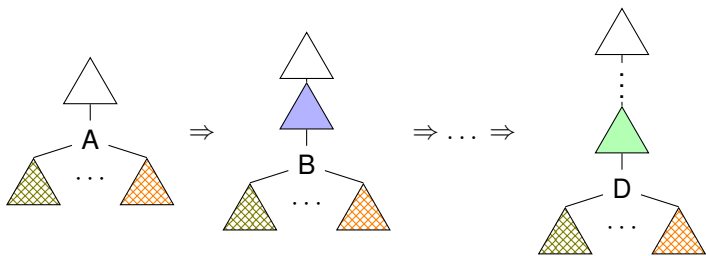




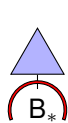
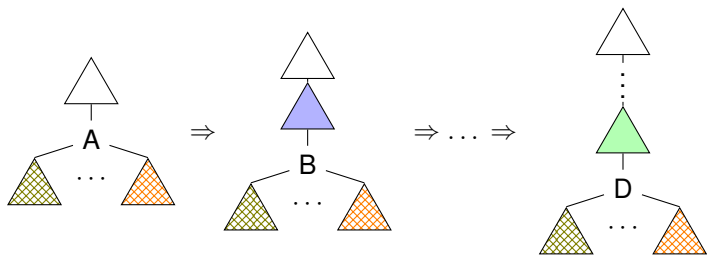






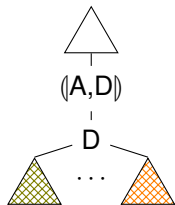
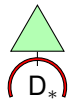
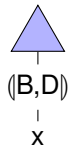


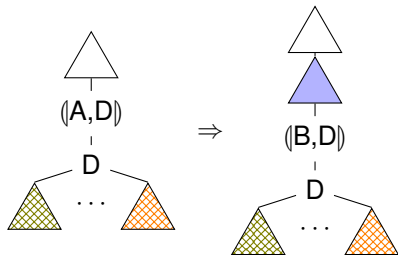
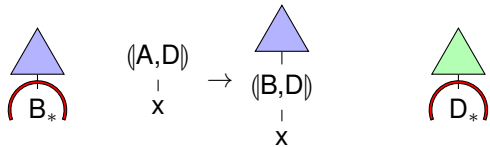
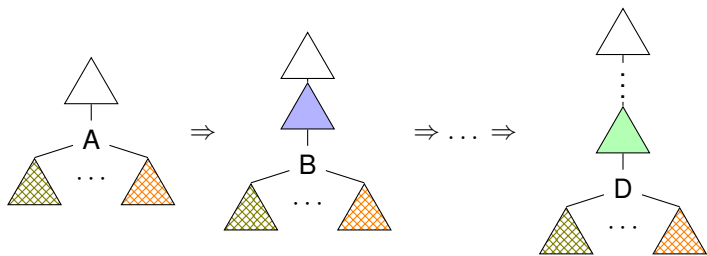


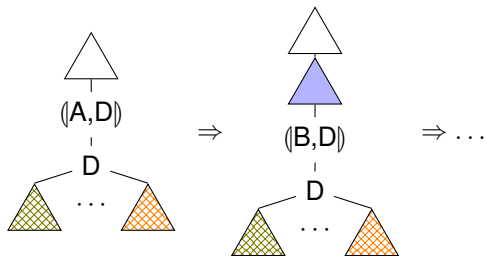
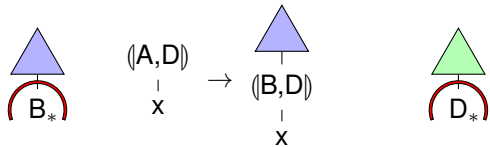
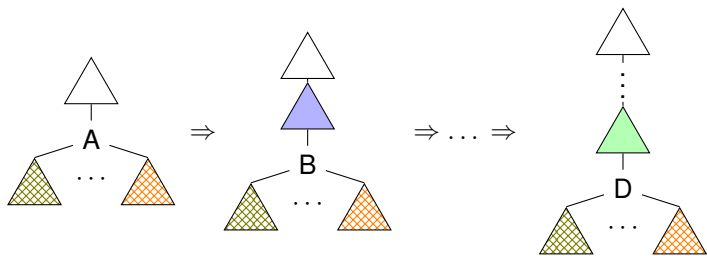


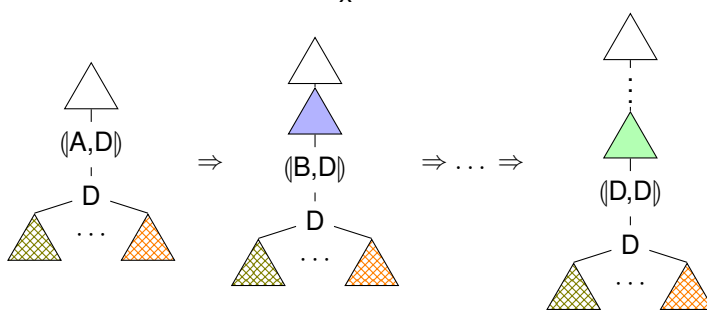
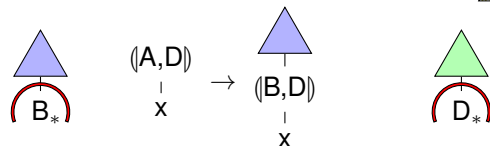
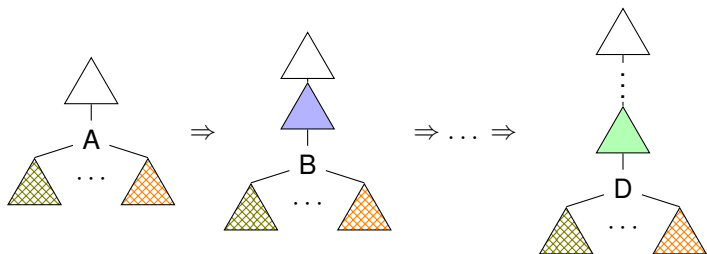
$(A, D)$   
|  
x

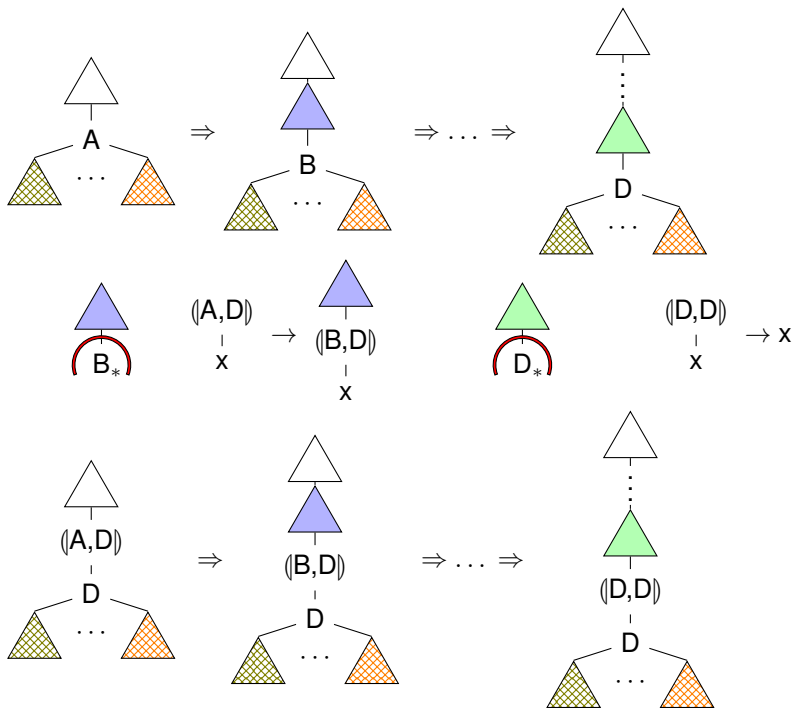
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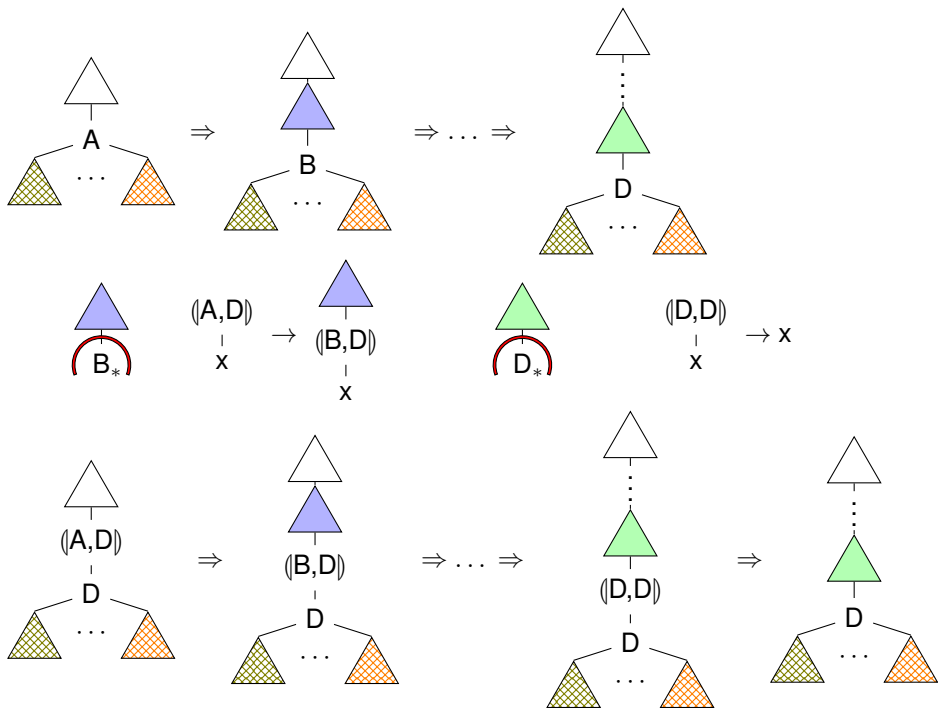






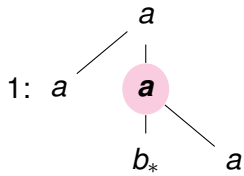






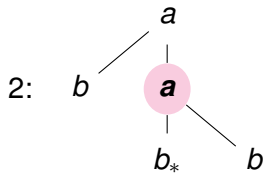
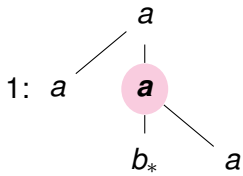
$\mathcal{I}$ : **b**

$\mathcal{I}$ :  **$b$**

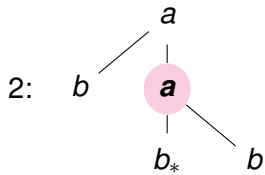
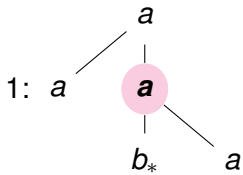




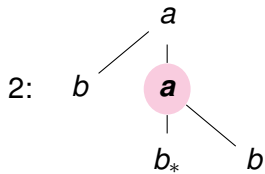
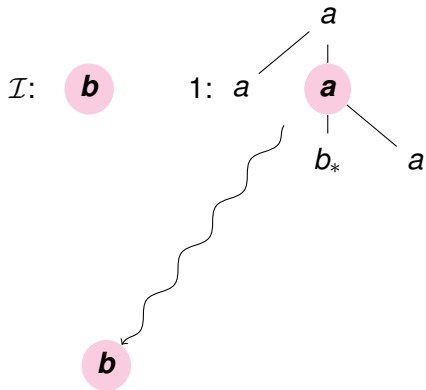
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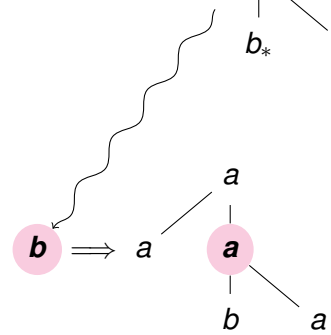
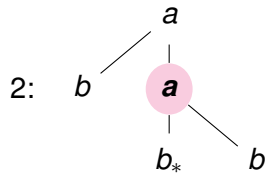
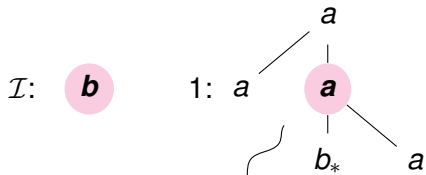


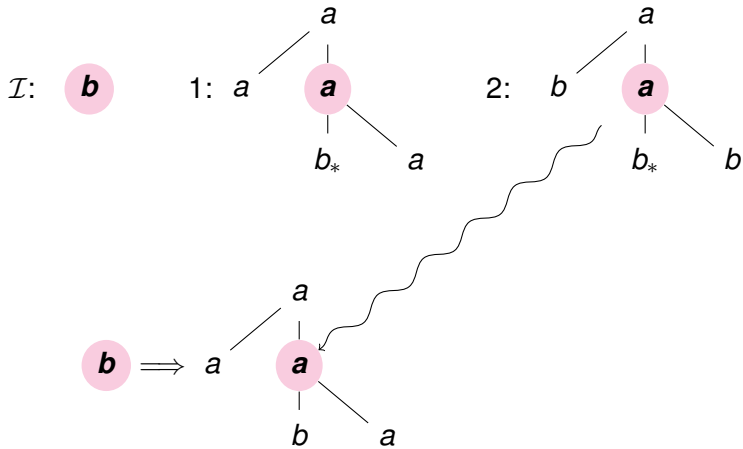
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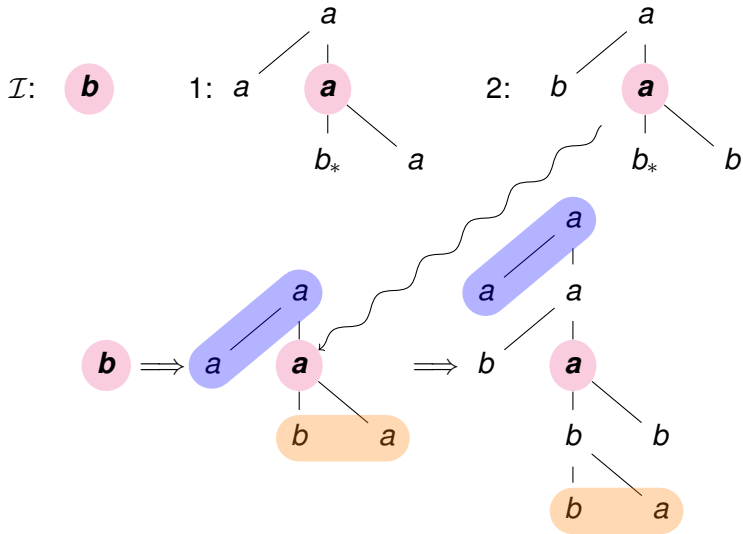


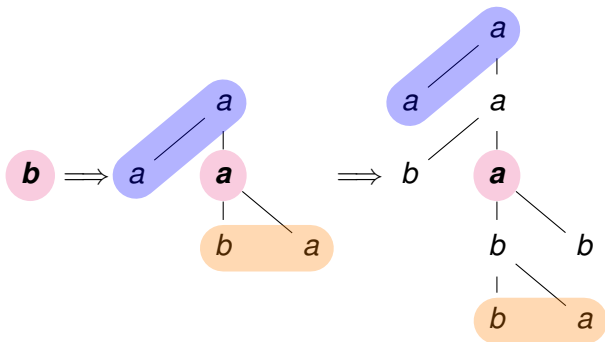
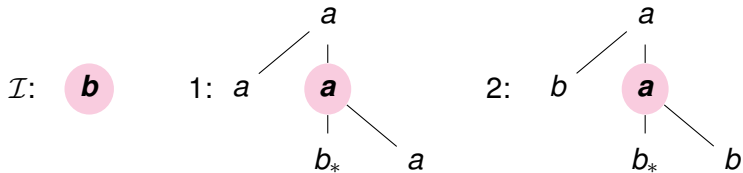
**b**





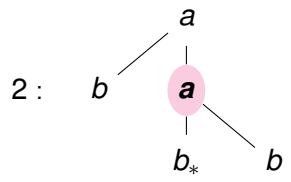
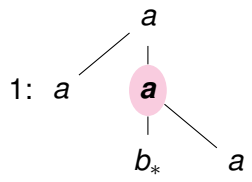






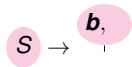
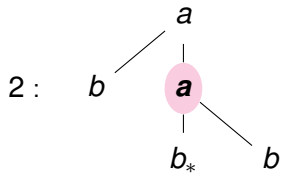
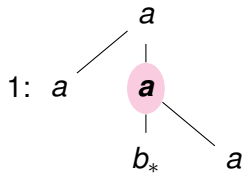
yield language:  $w b w$  for  $w \in \{a, b\}^*$

$\mathcal{I}$ : **b**

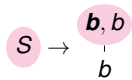
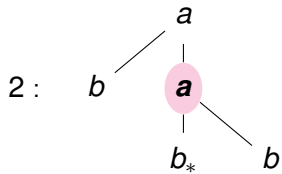
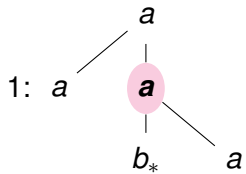




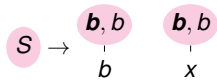
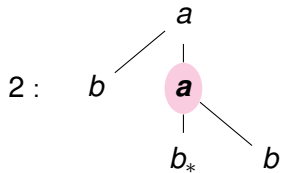
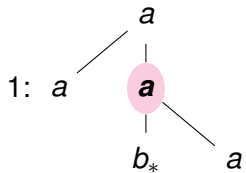
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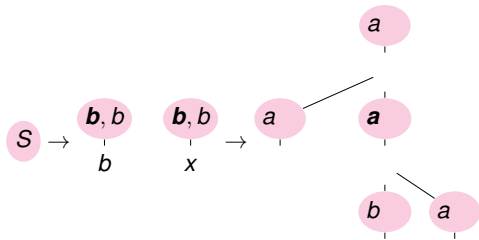
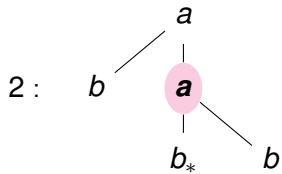
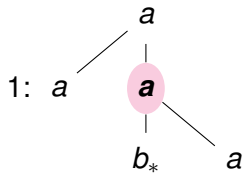
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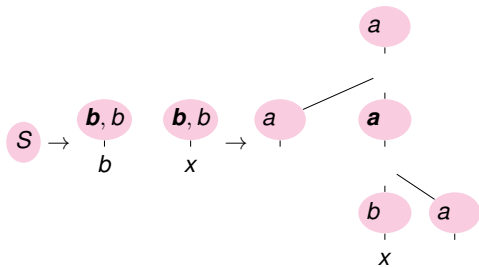
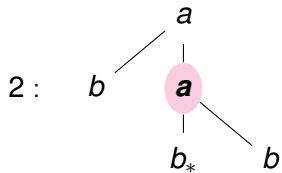
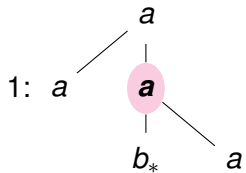
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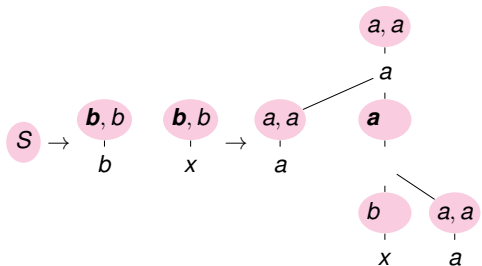
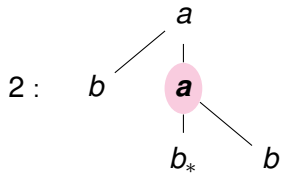
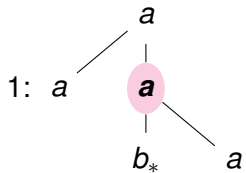
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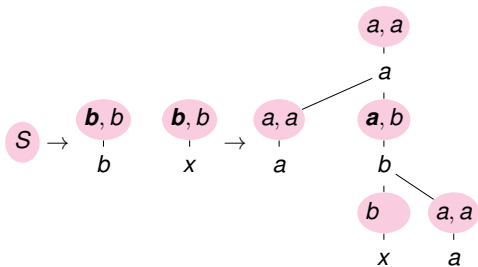
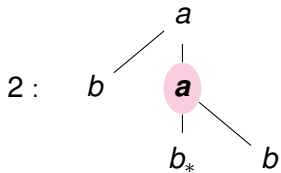
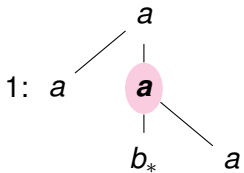
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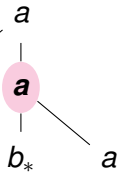
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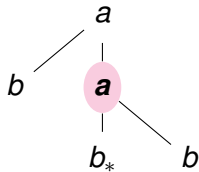
$\mathcal{I}$ :



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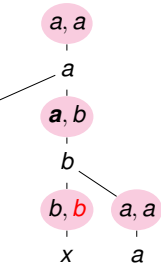
2:  $b$



$S \rightarrow$



$\rightarrow$

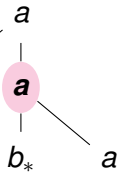




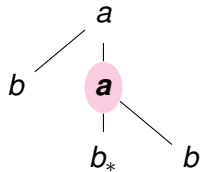
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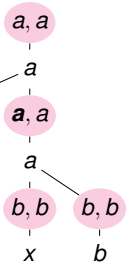
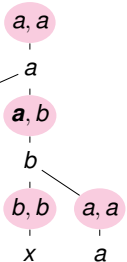
1:  $a$



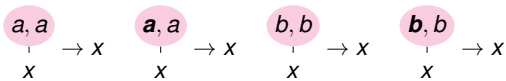
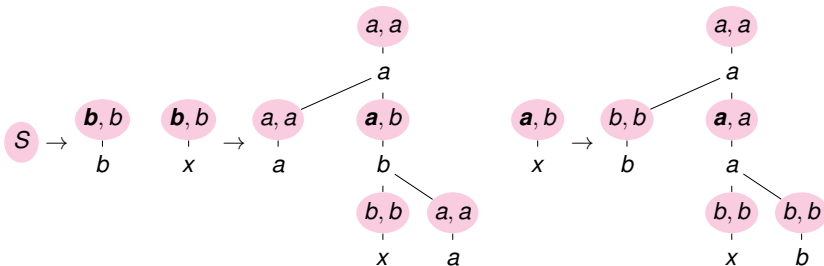
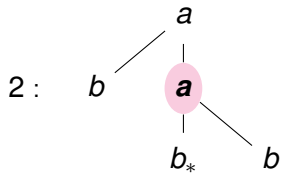
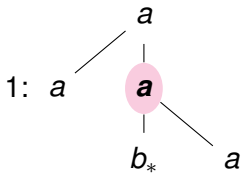
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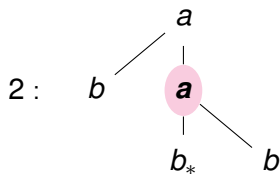
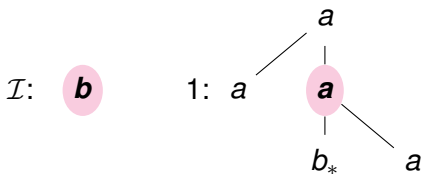


$S \rightarrow$



$\mathcal{I}$ : **b**





**a, a**

**a, a**

guessing:

one-state nondeterministic top-down tree transducer

**b, b**

**a, a**

**b, b**

**b, b**

$x$

$a$

$x$

$b$

**a, a**

**a, a**

**b, b**

**b, b**

$x$

$x$

$x$

$x$

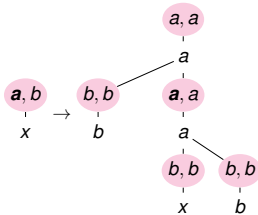
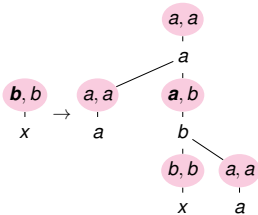
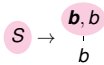
$\rightarrow x$

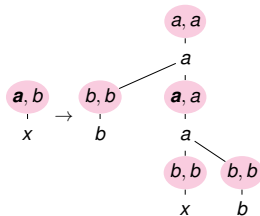
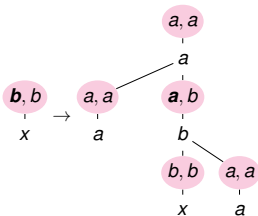
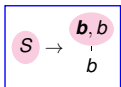
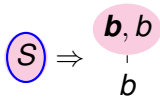
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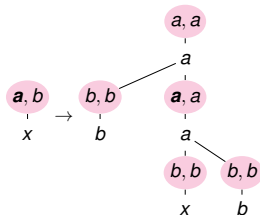
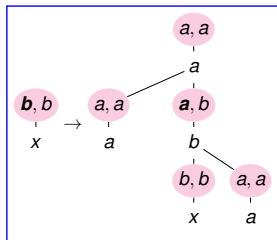
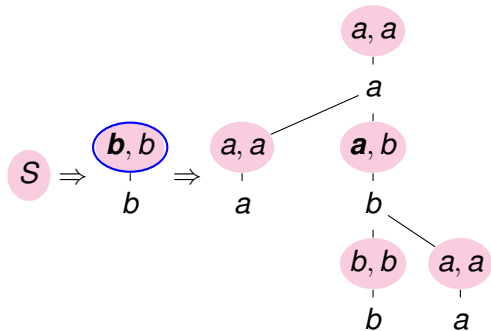
$\rightarrow x$

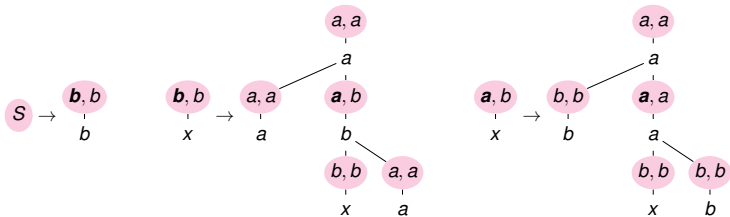
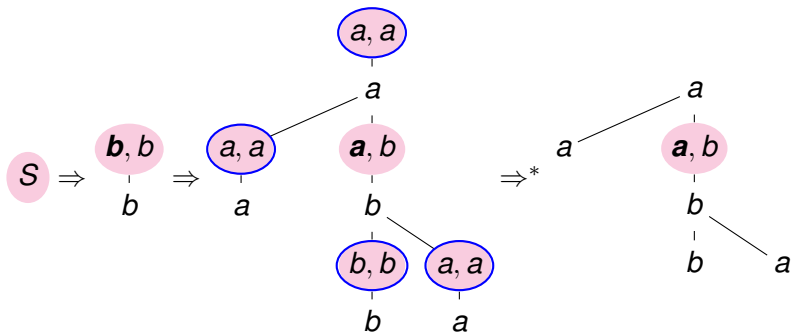
$\rightarrow x$

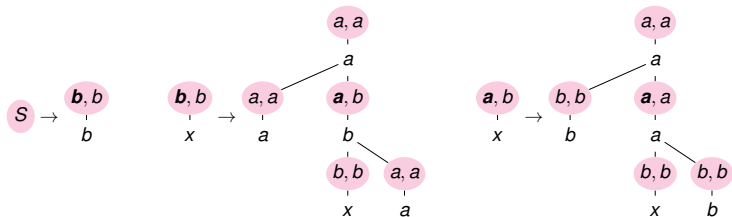
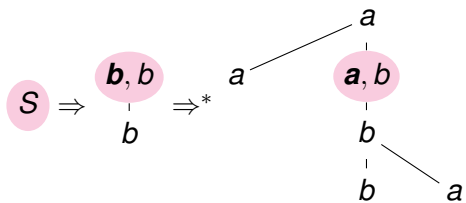
S



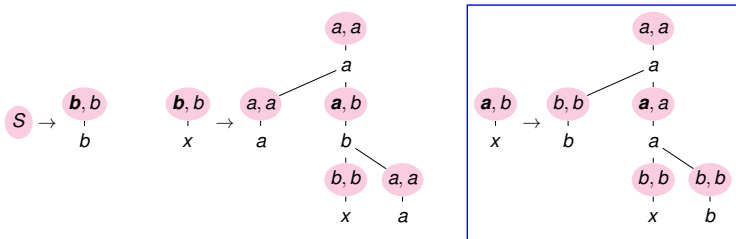
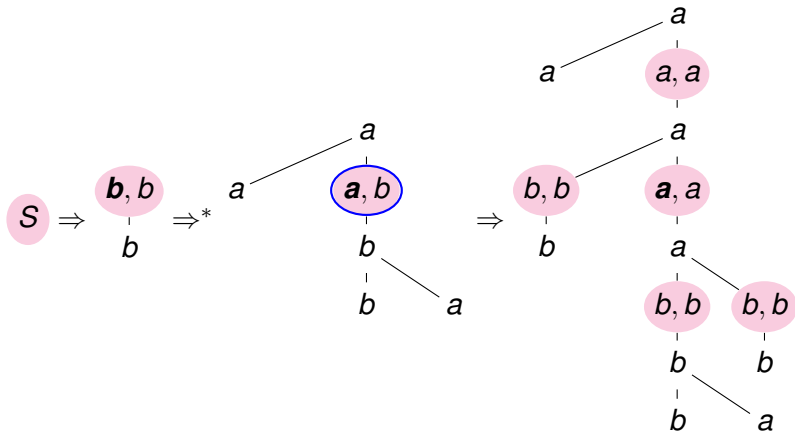


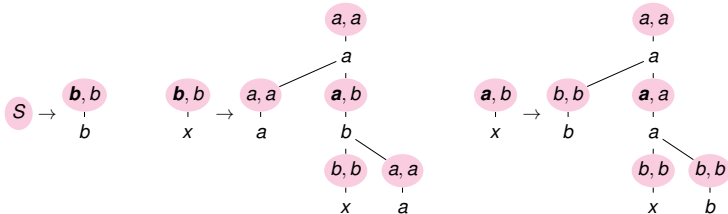
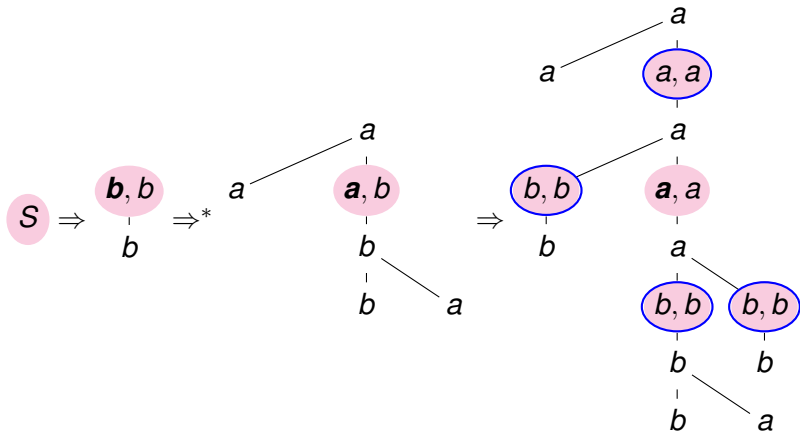


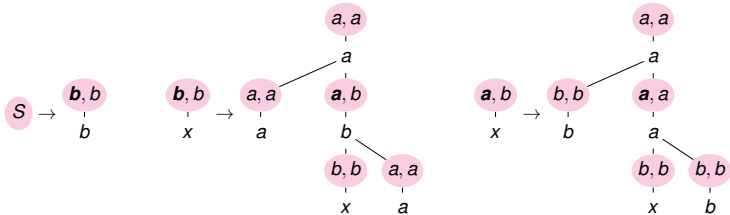
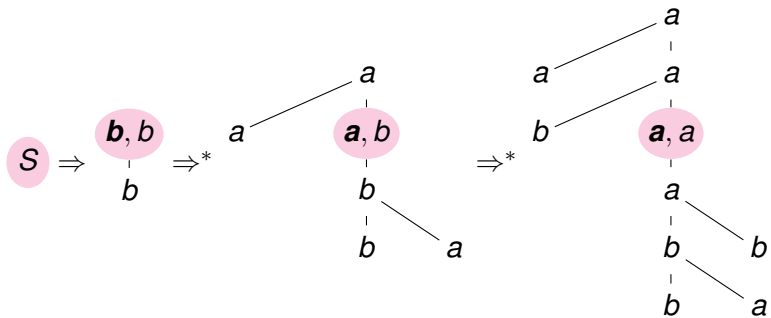


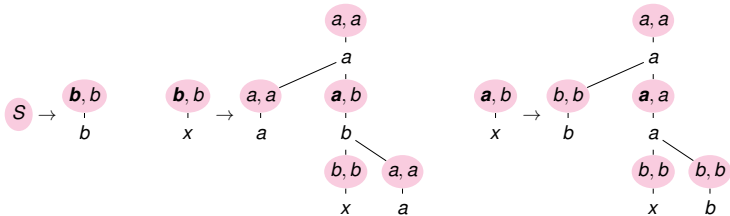
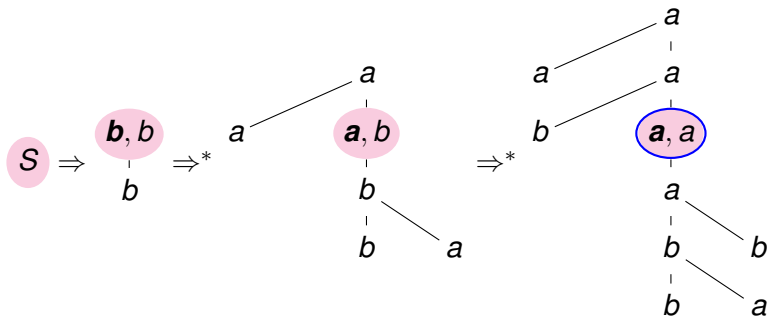


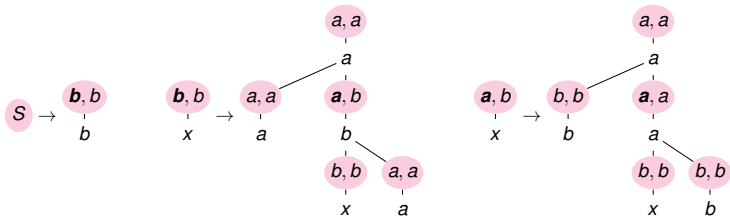
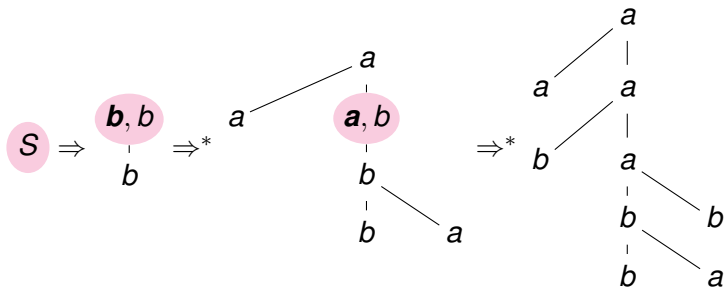












## conclusions

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