Weighted Symbolic Automata with Data Storage

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Technische Universität Dresden

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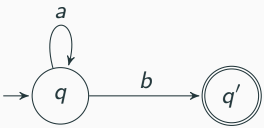
Automata over Infinite Alphabets



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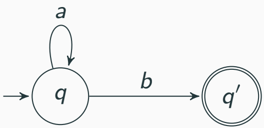
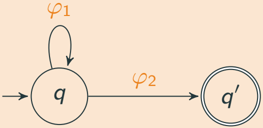


$d_1 \ldots d_n \in D^*$



symbolic automata [Veanes et al. 10]
+ many extensions

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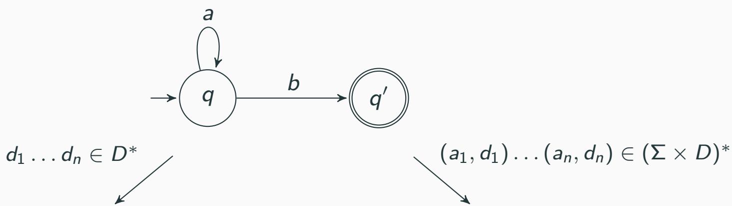

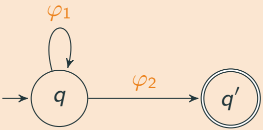

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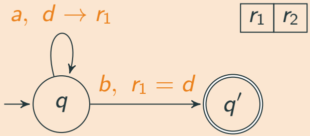

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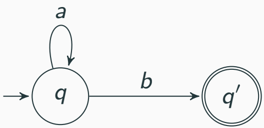

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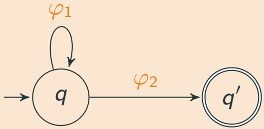

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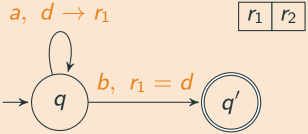

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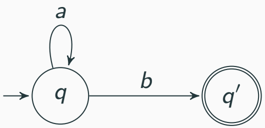

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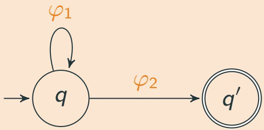

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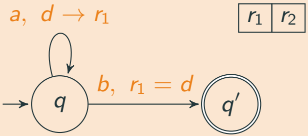

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weighted symbolic automata with data storage

Content

The Automaton Model: Automata with Storage

Unital Valuation Monoid

[Droste, Meinecke 12]

$(K, +, \mathrm{val}, 0, 1)$

- $(K, +, 0)$ commutative monoid
- $\mathrm{val}\colon K^* \to K$
  - $\mathrm{val}(a) = a$
  - $\mathrm{val}(\dots 0 \dots) = 0$
  - $\mathrm{val}(\dots 1 \dots) = \mathrm{val}(\dots\dots)$ ignore "1"s
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Example

$K_{\mathrm{avg}} = (\mathbb{R} \cup \{-\infty, \mathbb{1}\}, \sup, \mathrm{avg}, -\infty, \mathbb{1})$

$$\mathrm{avg}(k_1 \ldots k_n) = \frac{1}{n} \cdot (k_1 + \ldots + k_n)$$

Storage Type [Scott 67, Engelfriet 86]

$S = (C, \quad P, F, c_0)$

- C set (configurations)
- P set of functions $p\colon C \quad \to \{\text{true}, \text{false}\}$ (predicates)
- F set of partial functions $f\colon C \quad \to C$ (instructions)
- c_0 element of C (initial configuration)

Data Storage Type [Scott 67, Engelfriet 86]

$S = (C, D, P, F, c_0)$

- C set (configurations)
- D set (storage inputs)
- P set of functions $p\colon C \times D \to \{\text{true}, \text{false}\}$ (predicates)
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Example

$\textsc{Count} = (\mathbb{N}, \mathbb{N}, \{\top, 0?\}, \{+, -\}, 0)$

$A = (Q, \Pi, Q_0, Q_f, T, \mathrm{wt})$ (S, D, K)-automaton

- Q finite set (states)
  $Q_0 \subseteq Q$ (initial states), $Q_f \subseteq Q$ (final states)
- (D, Π) label structure
- T finite set (transitions)

$$\tau = (q_1, \pi, p, q_2, f)$$

$q_1, q_2 \in Q, \quad \pi \in \Pi, \quad p \in P, \quad f \in F$

- $\mathrm{wt}\colon T \times D \to K$ (weight assignment)

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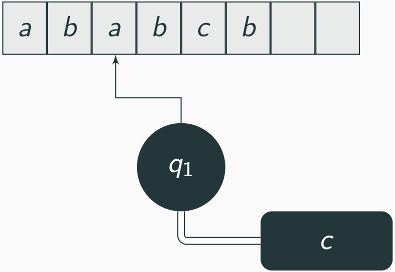

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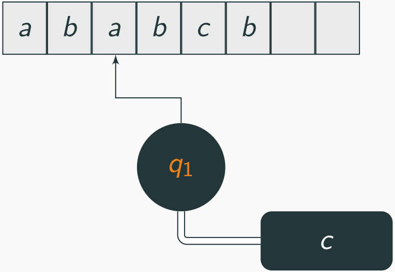

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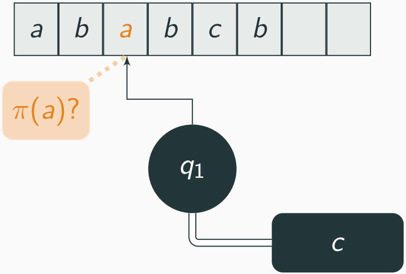

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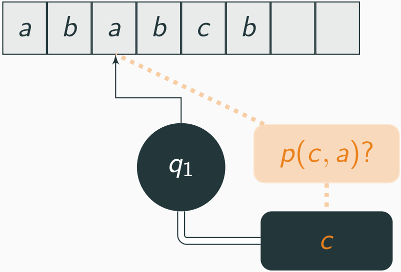

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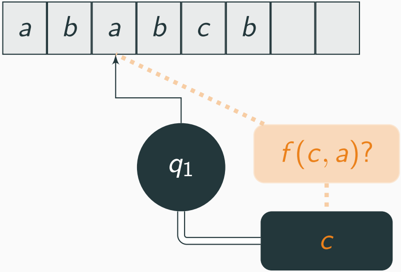

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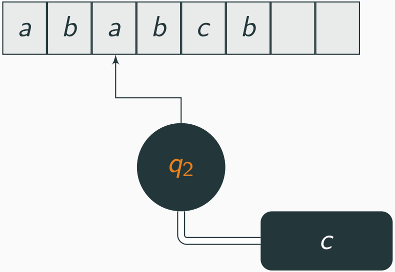

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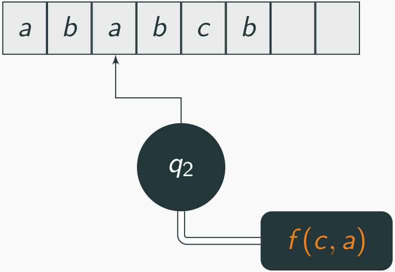

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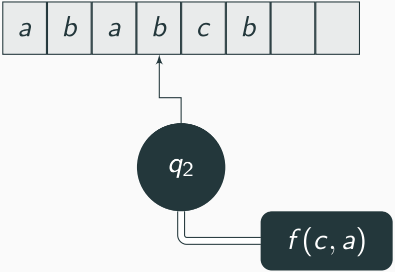

$w = d_1 \dots d_n \in \Sigma^*$
$\theta = \tau_1 \dots \tau_n, \quad \tau_i \in T$
$q_0 \in Q_0,\ q_f \in Q_f$

computation:

$$(q_0, d_1 d_2 \dots d_n, c_0) \vdash^{\tau_1} (q_1, d_2 \dots d_n, c_1) \vdash^{\tau_2} \dots \vdash^{\tau_n} (q_f, \varepsilon, c_n)$$

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$\Theta(w)$

weighted language recognized by A: $\|A\| \colon D^* \to K$

$$\|A\|(w) = \sum_{\theta \in \Theta(w)} \mathrm{wt}(\theta)$$

$$\mathrm{wt}(\tau_1 \dots \tau_n) = \mathrm{val}(\mathrm{wt}(\tau_1, d_1) \dots \mathrm{wt}(\tau_n, d_n))$$

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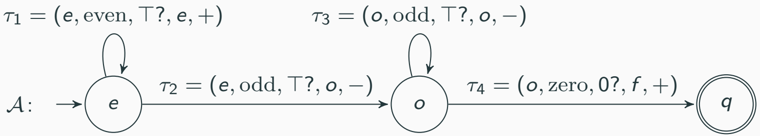

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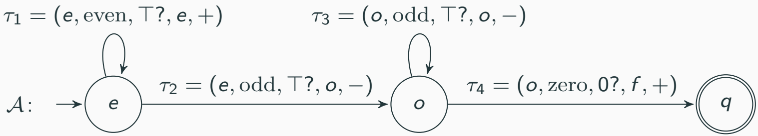

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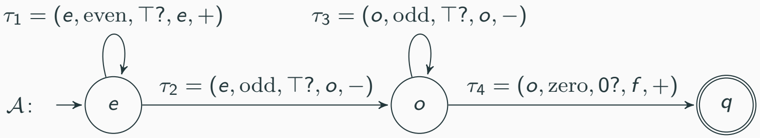

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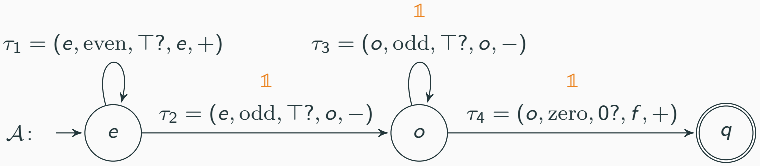

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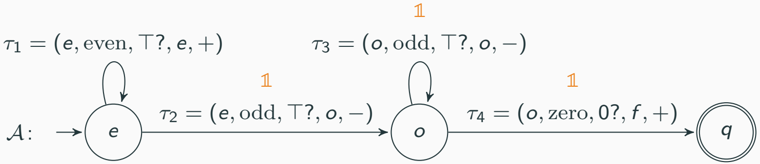

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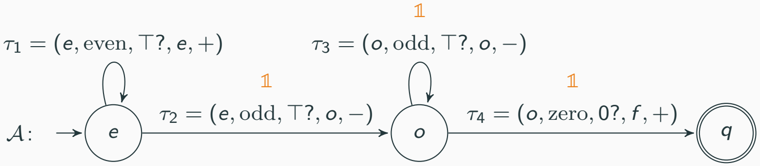

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$$[\![\mathcal{A}]\!](284770) = \mathrm{avg}(2 \; \mathbb{1} \; 4 \; \mathbb{1} \; \mathbb{1} \; \mathbb{1}) = \mathrm{avg}(2 \; 4) = 3$$

A Storage Type for Symbolic Visibly PD Automata

Symbolic Visibly Pushdown Automata [D'Antoni, Alur 14]

$$\Delta = \Sigma_{\langle} \cup \Sigma \cup \Sigma_{\rangle}$$

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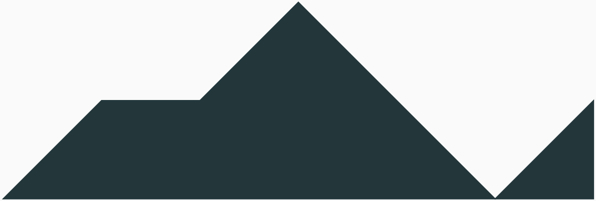

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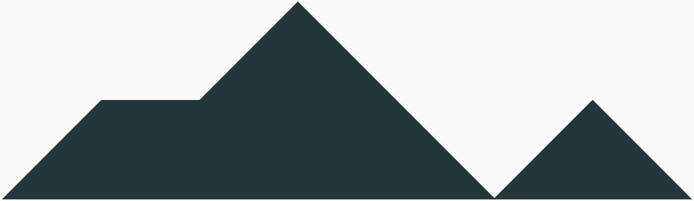

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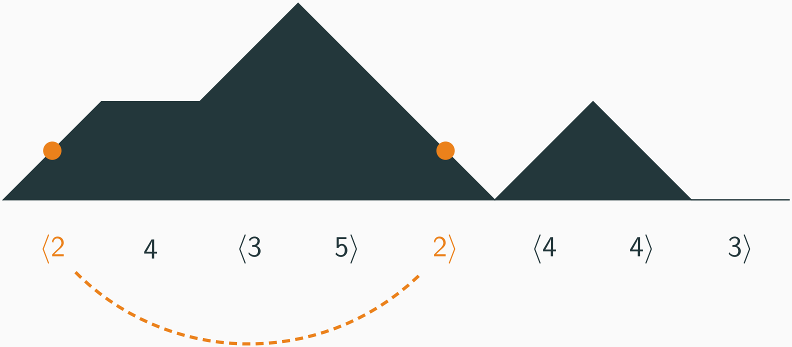

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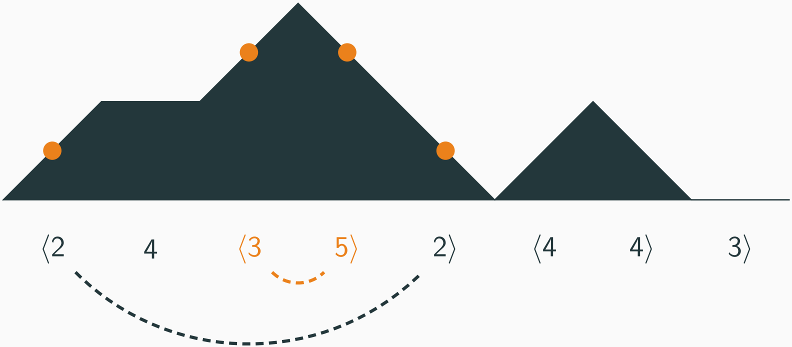

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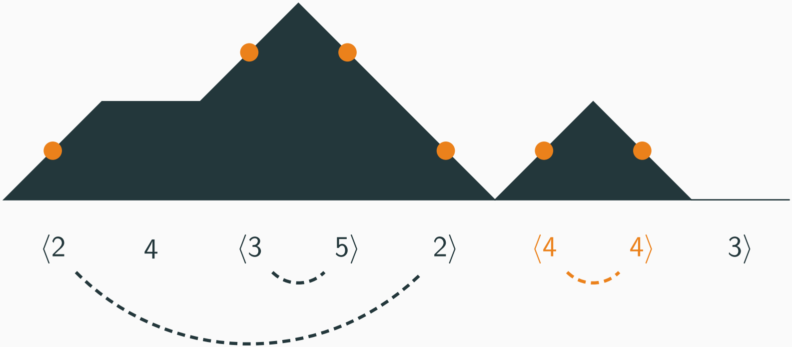

Symbolic Visibly Pushdown Automata [D'Antoni, Alur 14]

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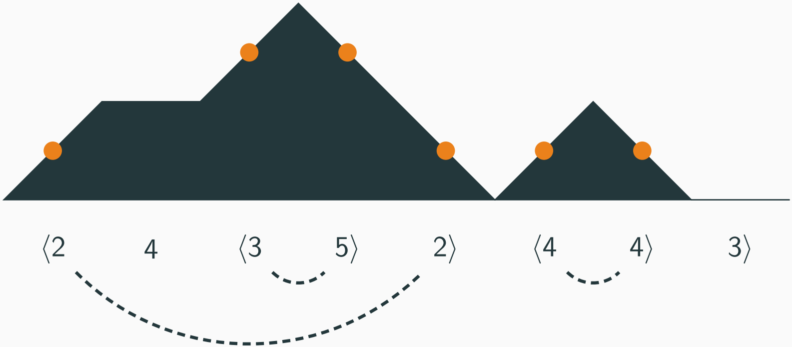

$(\langle x, y \rangle)$ matching $\wedge$ x even $\rightarrow x = y$

Data Storage Type $\mathrm{VP}(\Delta)$

$\mathrm{VP}(\Delta) = (C, \Delta, P, F, \varepsilon)$

- $C = (\Gamma \times \Sigma_\langle)^*$ $\Gamma \ldots$ pushdown symbols
- $P = \{\top\}$
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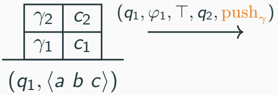

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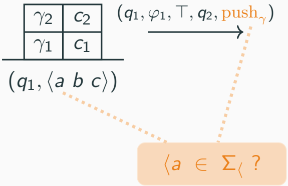

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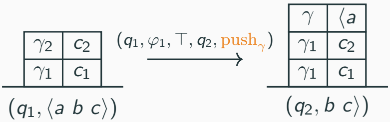

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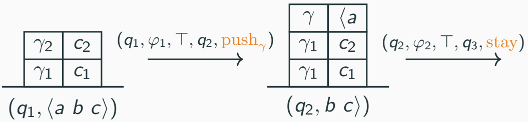

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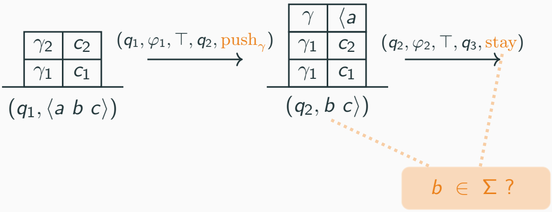

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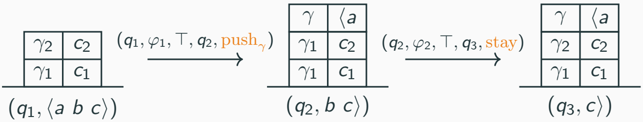

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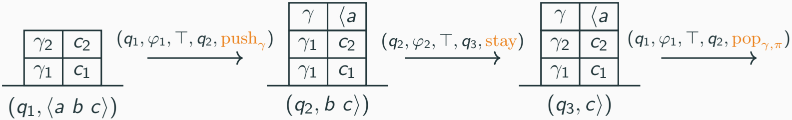

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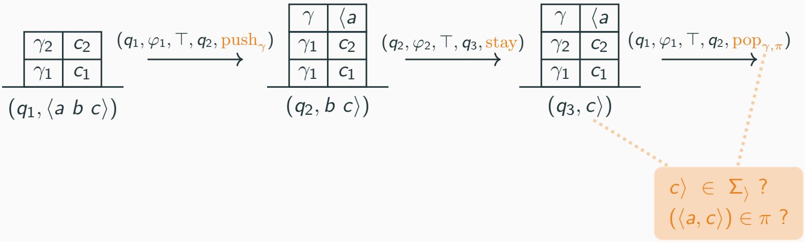

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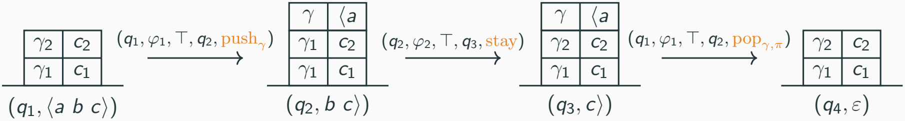

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Example

$(\langle x, y \rangle)$ **matching** $\wedge$ x **even** $\rightarrow x = y$

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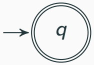

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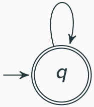
Example

$(\langle x, y \rangle)$ **matching** $\wedge$ x **even** $\rightarrow x = y$

$\tau_1 = (q, \mathrm{even}, \top, q, \mathrm{push}_e)$
$\tau_2 = (q, \mathrm{odd}, \top, q, \mathrm{push}_o)$



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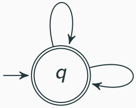

$$\tau_3 = (q, \top, \top, q, \mathrm{pop}_{e,\sim})$$
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Example $\qquad$ $(\langle x, y \rangle)$ matching $\wedge$ x even $\rightarrow x = y$

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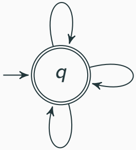

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$\tau_5 = (q, \top, \top, q, \mathrm{stay})$

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Theorem

Each $L \subseteq \Delta^*$ is recognizable by a symbolic visibly pd automaton iff it is recognizable by a $(\mathrm{VP}(\Delta), \Delta)$-automaton.

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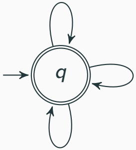

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$K_{\mathrm{avg}}: \quad w \mapsto \mathrm{avg}\{$even call symbols of $w\}$

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Theorem — **Emptiness of Support**

Let K be a zero-sum-free commutative strong bimonoid with decidable ZGP and $r\colon \Delta^* \to K$ $(\mathrm{VP}(\Delta), \Delta, K)$-recognizable.
It is decidable whether $\mathrm{supp}(r) = \emptyset$.

A Storage Type for Weighted Timed Automata

Semiring-Weighted Timed Automata $(K, +, \cdot, 0, 1)$

timed word $(a_1, t_1) \dots (a_n, t_n) \in (\Sigma \times \mathbb{R}_{\geq 0})^+$

Semiring-Weighted Timed Automata $(K, +, \cdot, 0, 1)$

timed word $(a\,,1)\;(b\,,3)\;(a\,,2)$

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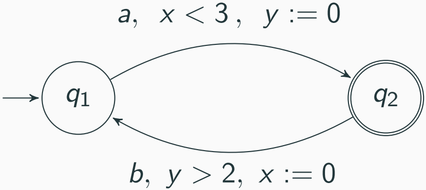
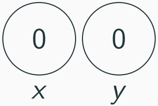
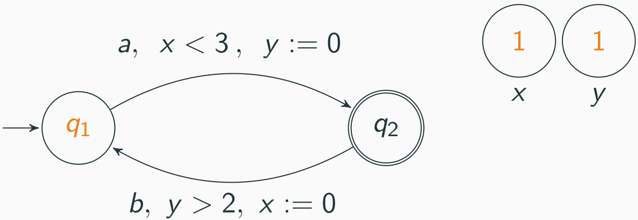




timed word $(a, 1)\ (b, 3)\ (a, 2)$



Semiring-Weighted Timed Automata

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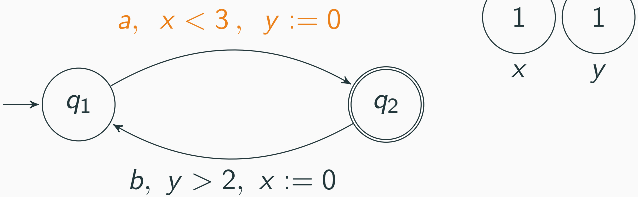

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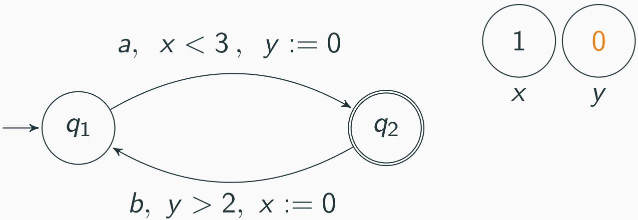

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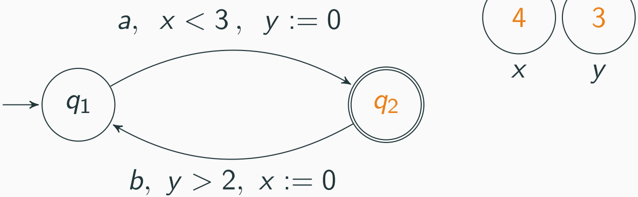

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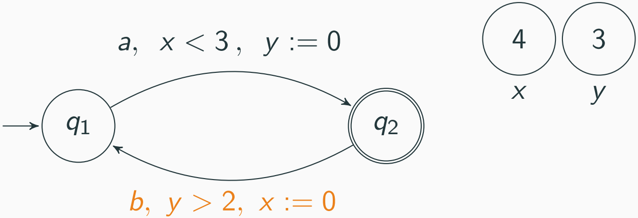

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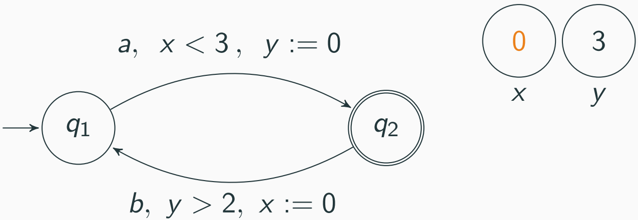

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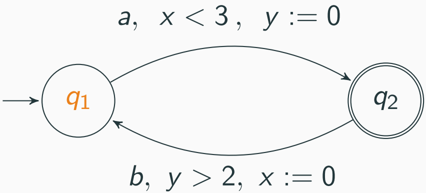

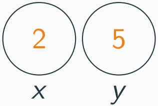

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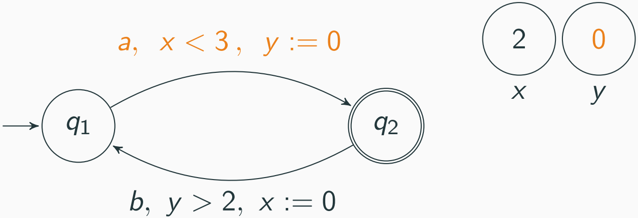

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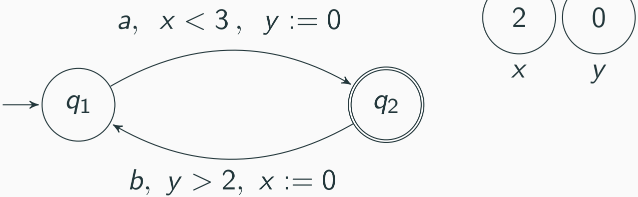

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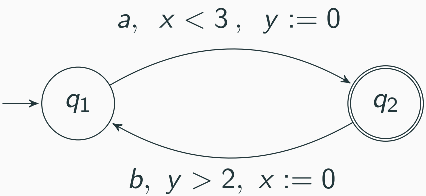

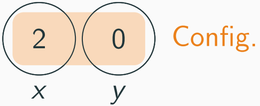

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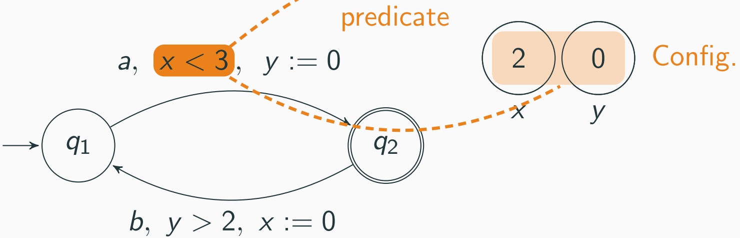

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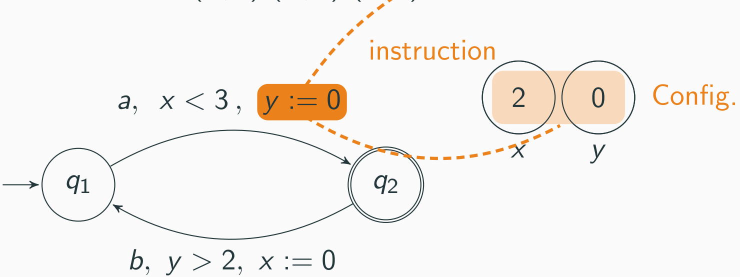

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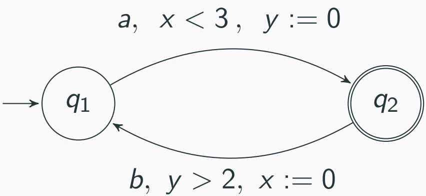

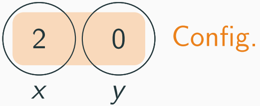

Theorem

Each $r\colon (\Sigma \times \mathbb{R}_{\geq 0})^+ \to K$ is recognizable by a weighted timed automaton iff it is $(\mathrm{Clock}, \Sigma \times \mathbb{R}_{\geq 0}, K)$-recognizable.

A Logical Characterization

Weighted symbolic MSO-logic with data storage [VogDroHer16]

$\sum_B e$ — (S, D, K)-expression

B storage behavior $(p_1, f_1) \ldots (p_n, f_n)$

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$e ::= \mathrm{Val}_\kappa \mid (e + e) \mid (\varphi \triangleright e) \mid \sum_x e \mid \sum_X e$ [FülStüVog10]

φ unweighted MSO-logic with $P_\pi(x)$

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Theorem **Büchi result**

Each $r \colon D^* \to K$ is definable by some (S, D, K)-expression iff it is (S, D, K)-recognizable.

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Corollary **Satisfiability problem**

The satisfiability problem of each $(\mathrm{VP}(\Delta), \Delta, K)$-expression is decidable if K is a zero-sum-free commutative strong bimonoid with decidable ZGP.

Thank you!

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References I

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Support of (S, D, K)-automata

Zero generation problem (ZGP) [Kirsten]

The ZGP for a monoid $(K, \cdot, 1)$ with zero 0 consists of two integers $m, n \in \mathbb{N}$, elements $k_1, \ldots, k_m, k'_1, \ldots, k'_n \in K$ and the question whether

$$0 \in k_1 \cdot \ldots \cdot k_m \cdot \langle k'_1, \ldots, k'_n \rangle.$$

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Theorem

Let K be a zero-sum-free commutative strong bimonoid.

1. For every (S, D, K)-recognizable language $r \colon D^* \to K$, $\mathrm{supp}(r)$ is (S, D)-recognizable.
2. Let $|\Sigma| \geq 2$. There is an effective construction of an (S, D)-automaton recognizing $\mathrm{supp}(\llbracket \mathcal{A} \rrbracket)$ from any given (S, D, K)-automaton $\mathcal{A}$ iff $(K, \cdot, 1)$ has a decidable ZGP.