Erratum regarding algorithms claimed to run in pseudo-polynomial time

May 5, 2022

1 Correction of statements

This erratum addresses claims in the following papers:

[BKKW17] Christel Baier, Joachim Klein, Sascha Klüppelholz, and Sascha Wunderlich. Maximizing the conditional expected reward for reaching the goal. In Axel Legay and Tiziana Margaria, editors, *Tools and Algorithms for the Construction and Analysis of Systems*, pages 269–285, Berlin, Heidelberg, 2017. Springer Berlin Heidelberg

[BBPS19] Christel Baier, Nathalie Bertrand, Jakob Piribauer, and Ocan Sankur. Long-run satisfaction of path properties. In *Proc. of 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, pages 1–14. IEEE, 2019

[PB20] Jakob Piribauer and Christel Baier. On Skolem-hardness and saturation points in Markov decision processes. In Artur Czumaj, Anuj Dawar, and Emanuela Merelli, editors, *Proc. of 47th International Colloquium on Automata, Languages, and Programming, (ICALP)*, volume 168 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 138:1–138:17. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2020

In these three papers, exponential-time algorithms are presented and falsely claimed to run in pseudo-polynomial time. More precisely, this concerns

- Theorem 2 in [BKKW17],
- Theorem IV.10 in [BBPS19], and
- Theorem 13 and Corollary 14 in [PB20].

All these statements can be corrected by replacing "pseudo-polynomial time" by "exponential time". These corrections do not affect the main contributions or any further results presented in these papers.

2 Explanation

2.1 Optimal conditional expectations in [BKKW17]

In [BKKW17], the optimization of conditional expected accumulated rewards under the condition that a goal state is reached in Markov decision processes (MDPs) is investigated. It is claimed [BKKW17, Theorem 2] that the threshold problem whether the optimal value exceeds a given rational bound is solvable in pseudo-polynomial time.

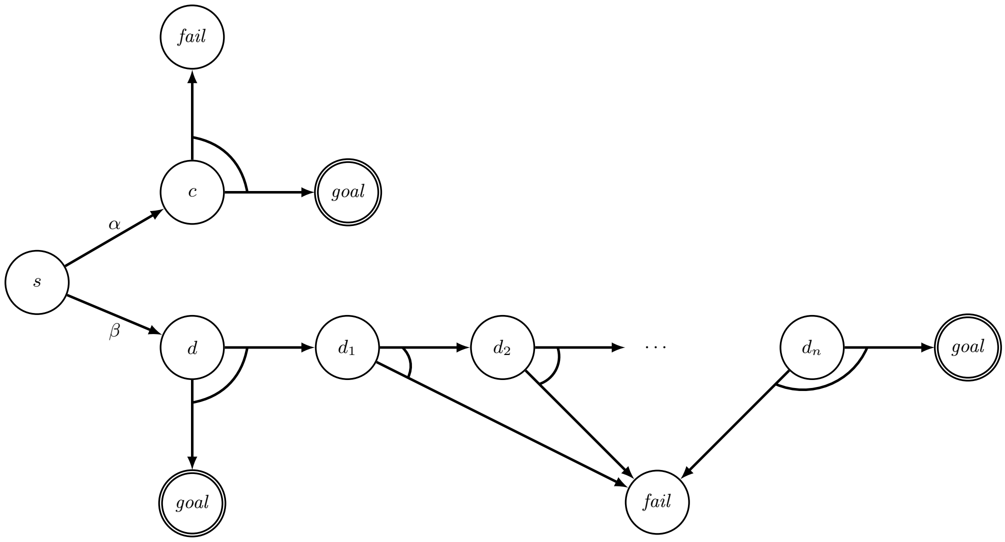

Figure 1: The MDP $\mathcal{M}_n$. All non-trivial transition probabilities are 1/2.

Given an MDP $\mathcal{M}$ with state space S, actions Act, and transition probability function P, the proposed algorithm computes a saturation point K that is a bound on the accumulated weight after which it is known how optimal schedulers behave. For the computation of the saturation point, the following values are defined for a given set of state $goal$, a state $s \notin goal$ and an action α:

- $p_s = \Pr^{\max}_{\mathcal{M},s}(\Diamond goal)$.
- $p_{s,\alpha} = \sum_{t \in S} P(s, \alpha, t) \cdot \Pr^{\max}_{\mathcal{M},t}(\Diamond goal)$.
- $Act^{\max}(s) = \{\alpha \in Act(s) \mid p_{s,\alpha} = p_s\}$.
- $\delta = \min\{p_s - p_{s,\alpha} \mid s \notin goal \text{ and } \alpha \notin Act^{\max}(s)\}$.

The saturation point K is then proportional to $\frac{1}{\delta}$. Afterwards, the algorithm repeats a loop K times. It is falsely claimed that K is of pseudo-polynomial time.

To see this, consider the MDP $\mathcal{M}_n$ for a natural number n depicted in Figure 1. All non-trivial transition probabilities are 1/2. The only non-deterministic choice is in the initial state s_0 between actions α and β. We observe the following reachability probabilities from states c and d (from which the process behaves purely probabilistic):

$$\Pr_{\mathcal{M},c}(\Diamond goal) = \frac{1}{2},$$
$$\Pr_{\mathcal{M},d}(\Diamond goal) = \frac{1}{2} + \frac{1}{2^{n+1}}.$$

For state s in $\mathcal{M}_n$, we conclude that $p_s = \frac{1}{2} + \frac{1}{2^{n+1}}$ while $p_{s,\alpha} = \frac{1}{2}$ where α is the only action not in $Act^{\max}(s)$. So, here $\delta \leq \frac{1}{2^{n+1}}$. This means that for the family of MDPs $\mathcal{M}_n$, the computed saturation points K grow exponentially in n and hence in the number of states. As all non-trivial transition probabilities are 1/2, a unary encoding of the numerical values does not change this and the algorithm does not run in pseudo-polynomial time.

2.2 Optimal long-run probabilities in [BBPS19]

In a labeled MDP $\mathcal{M}$, the long-run probability of a temporal formula φ under a scheduler $\mathfrak{S}$ is defined as the long-run average under $\mathfrak{S}$ of the probability that the suffixes of a run satisfy φ under $\mathfrak{S}$. In [BBPS19, Theorem IV.10], it is claimed that the optimal long-run probability for the constrained reachability formula $a \,\mathrm{U}\, b$ can be computed in polynomial time. The proposed algorithm computes a saturation point K similar to above, which is a bound on the number of consecutive visits to an a-state before the behavior of an optimal scheduler is known, and solves a mean-payoff problem on an MDP of size linear in K. The numerical value of K, however, is exponential, but not pseudo-polynomial, in the size of $\mathcal{M}$, as it relies on a minimal difference between reachability problems as above.

2.3 Model checking of certain frequency-LTL formulas in [PB20]

The results that the model-checking problem for certain frequency-LTL formulas can be solved in pseudo-polynomial time [PB20, Theorem 13, Corollary 14] is obtained by a modification of the algorithm for long-run probabilities of constrained reachability properties (see above). Hence, the same problem arises here and the proposed algorithms run in exponential, but not in pseudo-polynomial time.

References

[BBPS19] Christel Baier, Nathalie Bertrand, Jakob Piribauer, and Ocan Sankur. Long-run satisfaction of path properties. In *Proc. of 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, pages 1–14. IEEE, 2019.

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