Erratum regarding algorithms claimed to run in pseudo-polynomial time

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1 Correction of statements

This erratum addresses claims in the following papers:


In these three papers, exponential-time algorithms are presented and falsely claimed to run in pseudo-polynomial time. More precisely, this concerns

• Theorem 2 in [BKKW17],
• Theorem IV.10 in [BBPS19], and
• Theorem 13 and Corollary 14 in [PB20].

All these statements can be corrected by replacing “pseudo-polynomial time” by “exponential time”. These corrections do not affect the main contributions or any further results presented in these papers.

2 Explanation

2.1 Optimal conditional expectations in [BKKW17]

In [BKKW17], the optimization of conditional expected accumulated rewards under the condition that a goal state is reached in Markov decision processes (MDPs) is investigated. It is claimed [BKKW17, Theorem 2] that the threshold problem whether the optimal value exceeds a given rational bound is solvable in pseudo-polynomial time.
Given an MDP $\mathcal{M}$ with state space $S$, actions $\text{Act}$, and transition probability function $P$, the proposed algorithm computes a saturation point $K$ that is a bound on the accumulated weight after which it is known how optimal schedulers behave. For the computation of the saturation point, the following values are defined for a given set of state $\text{goal}$, a state $s \not\in \text{goal}$ and an action $\alpha$:

- $p_s = \text{Pr}_{\mathcal{M},s}(\Diamond \text{goal})$.
- $p_{s,\alpha} = \sum_{t \in S} P(s, \alpha, t) \cdot \text{Pr}_{\mathcal{M},t}(\Diamond \text{goal})$.
- $\text{Act}^{\max}(s) = \{ \alpha \in \text{Act}(s) \mid p_{s,\alpha} = p_s \}$.
- $\delta = \min\{p_s - p_{s,\alpha} \mid s \not\in \text{goal} \text{ and } \alpha \not\in \text{Act}^{\max}(s)\}$.

The saturation point $K$ is then proportional to $\frac{1}{\delta}$. Afterwards, the algorithm repeats a loop $K$ times. It is falsely claimed that $K$ is of pseudo-polynomial time.

To see this, consider the MDP $\mathcal{M}_n$ for a natural number $n$ depicted in Figure 1. All non-trivial transition probabilities are $1/2$. The only non-deterministic choice is in the initial state $s_0$ between actions $\alpha$ and $\beta$. We observe the following reachability probabilities from states $c$ and $d$ (from which the process behaves purely probabilistic):

$$\text{Pr}_{\mathcal{M},c}(\Diamond \text{goal}) = \frac{1}{2},$$

$$\text{Pr}_{\mathcal{M},d}(\Diamond \text{goal}) = \frac{1}{2} + \frac{1}{2^{2n+1}}.$$

For state $s$ in $\mathcal{M}_n$, we conclude that $p_s = \frac{1}{2} + \frac{1}{2^{2n+1}}$ while $p_{s,\alpha} = \frac{1}{2}$ where $\alpha$ is the only action not in $\text{Act}^{\max}(s)$. So, here $\delta \leq \frac{1}{2^{2n+1}}$. This means that for the family of MDPs $\mathcal{M}_n$, the computed saturation points $K$ grow exponentially in $n$ and hence in the number of states. As all non-trivial transition probabilities are $1/2$, a unary encoding of the numerical values does not change this and the algorithm does not run in pseudo-polynomial time.
2.2 Optimal long-run probabilities in [BBPS19]

In a labeled MDP $\mathcal{M}$, the long-run probability of a temporal formula $\varphi$ under a scheduler $\mathcal{S}$ is defined as the long-run average under $\mathcal{S}$ of the probability that the suffixes of a run satisfy $\varphi$ under $\mathcal{S}$. In [BBPS19, Theorem IV.10], it is claimed that the optimal long-run probability for the constrained reachability formula $a \mathbb{U} b$ can be computed in polynomial time. The proposed algorithm computes a saturation point $K$ similar to above, which is a bound on the number of consecutive visits to an $a$-state before the behavior of an optimal scheduler is known, and solves a mean-payoff problem on an MDP of size linear in $K$. The numerical value of $K$, however, is exponential, but not pseudo-polynomial, in the size of $\mathcal{M}$, as it relies on a minimal difference between reachability problems as above.

2.3 Model checking of certain frequency-LTL formulas in [PB20]

The results that the model-checking problem for certain frequency-LTL formulas can be solved in pseudo-polynomial time [PB20, Theorem 13, Corollary 14] is obtained by a modification of the algorithm for long-run probabilities of constrained reachability properties (see above). Hence, the same problem arises here and the proposed algorithms run in exponential, but not in pseudo-polynomial time.

References

